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Control Systems (2)

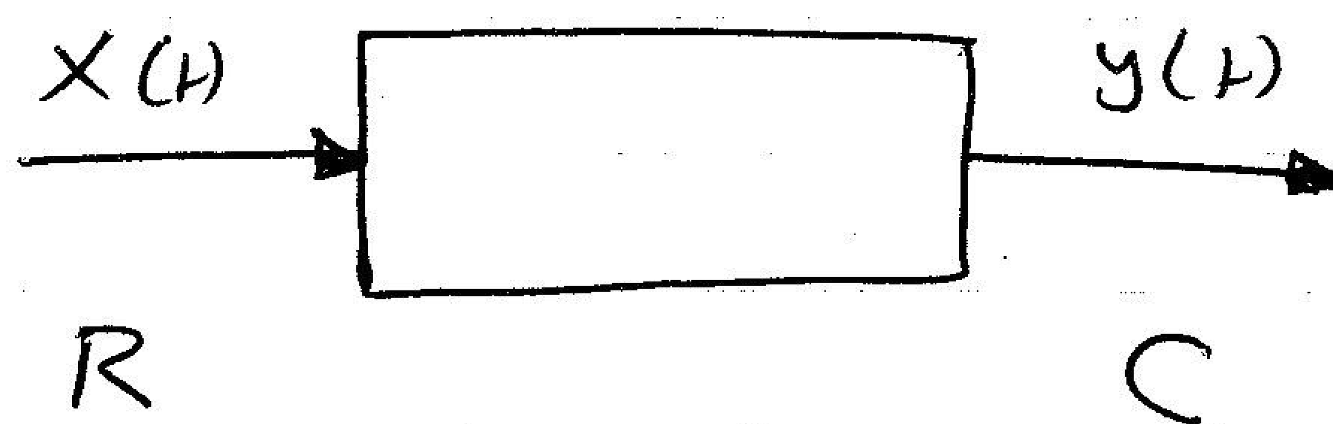
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RTN 11-12
ROOM 240

1. 1st Order & 2nd order Systems

First order systems: the system that its response rates with the change of its response.

and descriped by the following derivative equation:-

$$\tau y' + y = kx$$



$$y' = \frac{dy}{dt}$$

$$\tau \dot{y} + y = kx$$

τ : delay time (Time Constant).

$y(t)$: output - response.

$x(t)$: input

k : static gain (DC gain)

$$\tau Y(s)\, s + Y(s) = kX(s)$$

$$Y(s)\,[\tau s + 1] = kX(s)$$

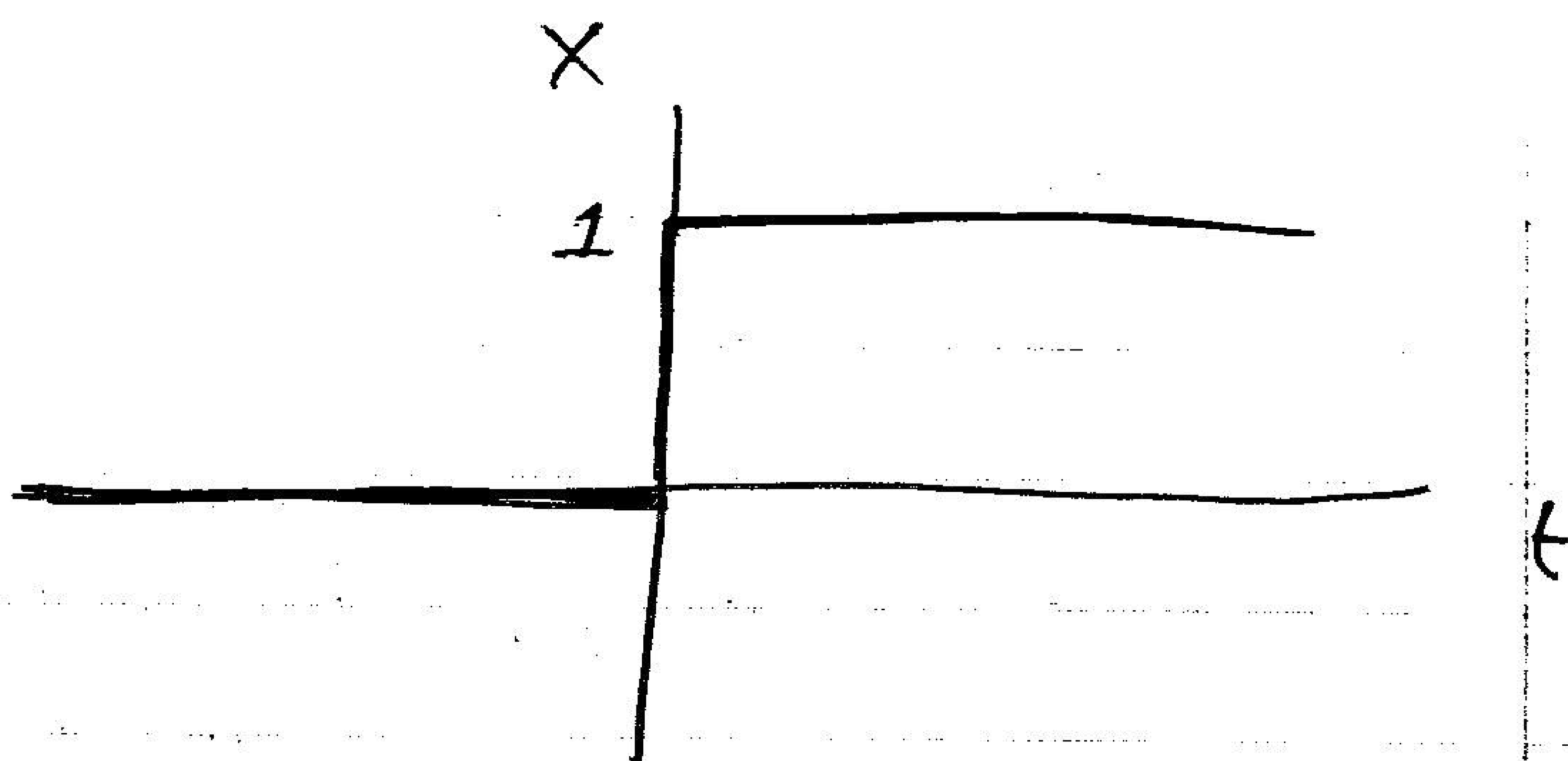
$$\text{Transfer function} = \frac{\text{output}}{\text{input}} = \frac{Y(s)}{X(s)} = \frac{k}{\tau s + 1}$$

$$Y(s) = \frac{k}{\tau s + 1} X(s)$$

$$X(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

↓

Step input



* If $x(t) = 1$ for $t > 0$

$$X(s) = \frac{1}{s}$$

$$Y(s) = \frac{k}{\tau s + 1} * \frac{1}{s}$$

$$\frac{k}{\tau s + 1} * \frac{1}{s} = \left(\frac{a}{\tau s + 1} + \frac{b}{s}\right) * s$$

Multiply by (s) then Made s = 0

$$\cancel{\frac{a s}{\tau s + 1}} + b \Rightarrow \frac{k}{\tau s + 1} = b \Rightarrow \boxed{b = k}$$

$\tau s + 1$

↓

$\boxed{s = \frac{-1}{\tau}}$

$$\Rightarrow \frac{k}{s} = a \Rightarrow a = -k\tau$$

$$\therefore Y(s) = \frac{-k\tau}{\tau s + 1} + \frac{k}{s}$$

$$Y(s) = k\left[\frac{1}{s} + \frac{\tau}{\tau s + 1}\right]$$

$\mathcal{L}^{-1} \quad y(t) = k\left[1 - e^{-t/\tau}\right]$

$\frac{\tau}{\tau s+1} = \frac{1}{s+\frac{1}{\tau}}$

$= e^{-t/\tau}$

$e^{at} = \frac{1}{s-a}$

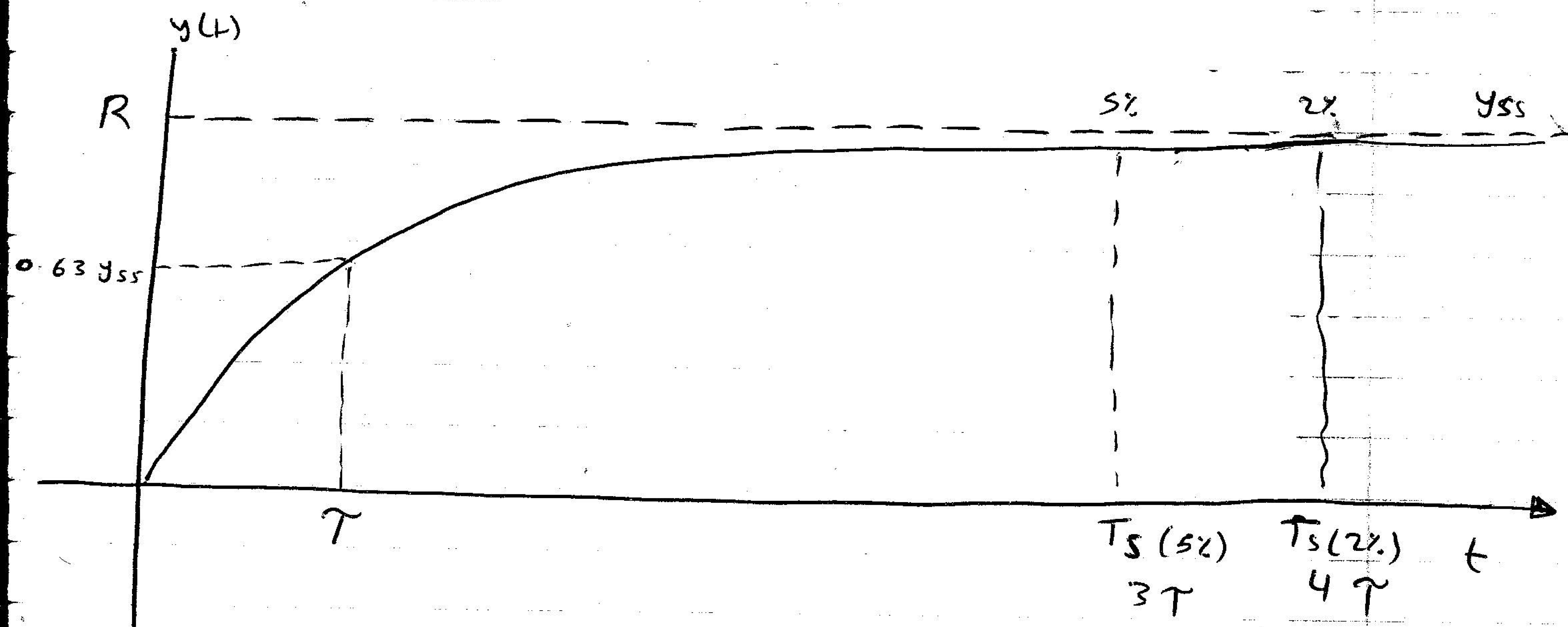
$e^{-at} = \frac{1}{s+a}$

∴ Response equation for unit step input:

$y(t) = k\left[1 - e^{-t/\tau}\right]$

$y(t) = KR\left[1 - e^{-t/\tau}\right]$

for unit step input $x(t) = R$



Static error = $R(t) - y(t) \Big|_{t=\infty}$

★ y_{ss} final value of response given at $t = \infty$

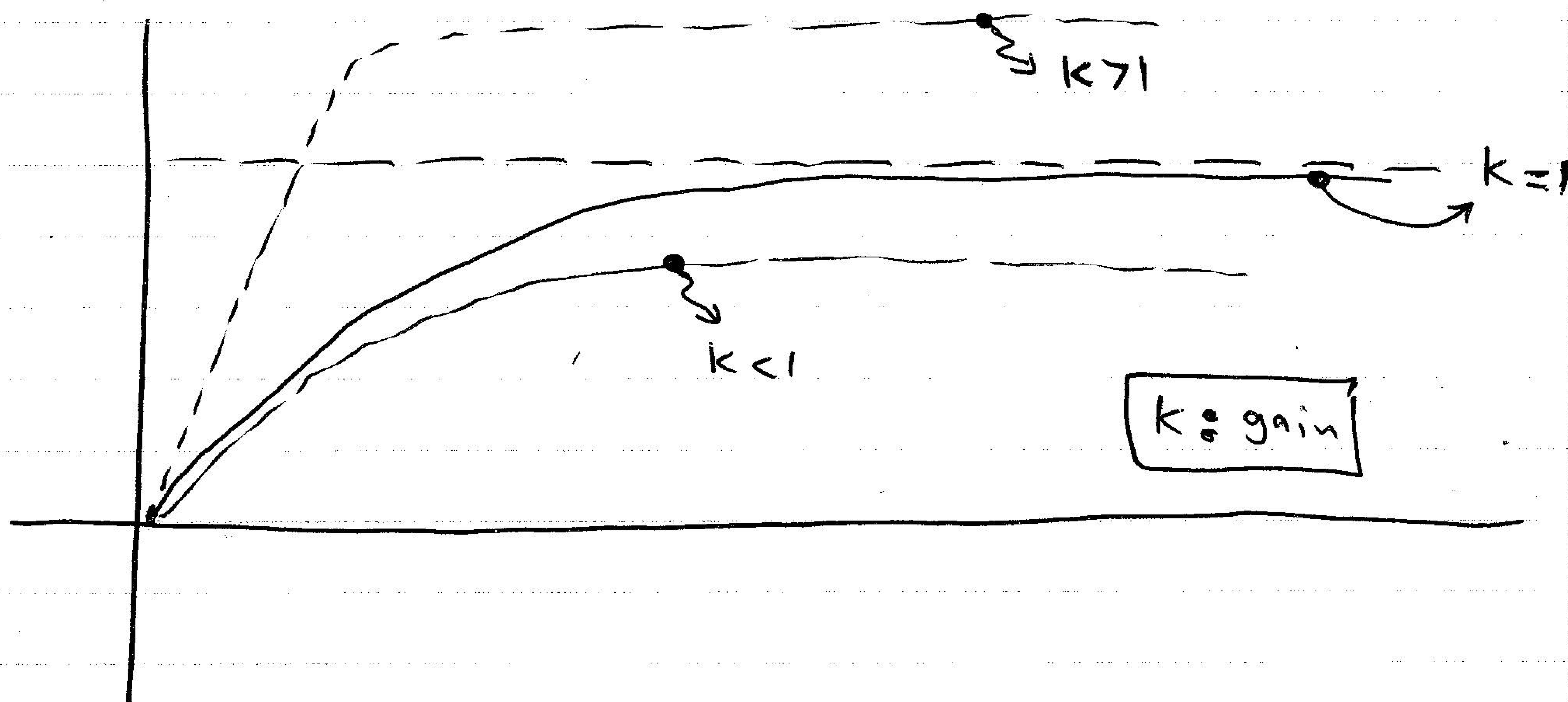
$y(t) = KR(1 - e^{-t/\tau})$

$y(\infty) = KR$

$$Y(s) = \frac{k}{\tau s + 1} * \frac{R}{s}$$

$$Y(\infty) = \lim_{s \to 0} s\, Y(s)$$

$$= \lim_{s \to 0} \frac{s\, kR}{\tau s + 1} * \frac{1}{s} = \boxed{kR}$$



$$y(\tau) = kR(1 - e^{-\frac{\tau}{\tau}}) = kR(1 - 0.37)$$

$$= \boxed{0.63\, kR}$$

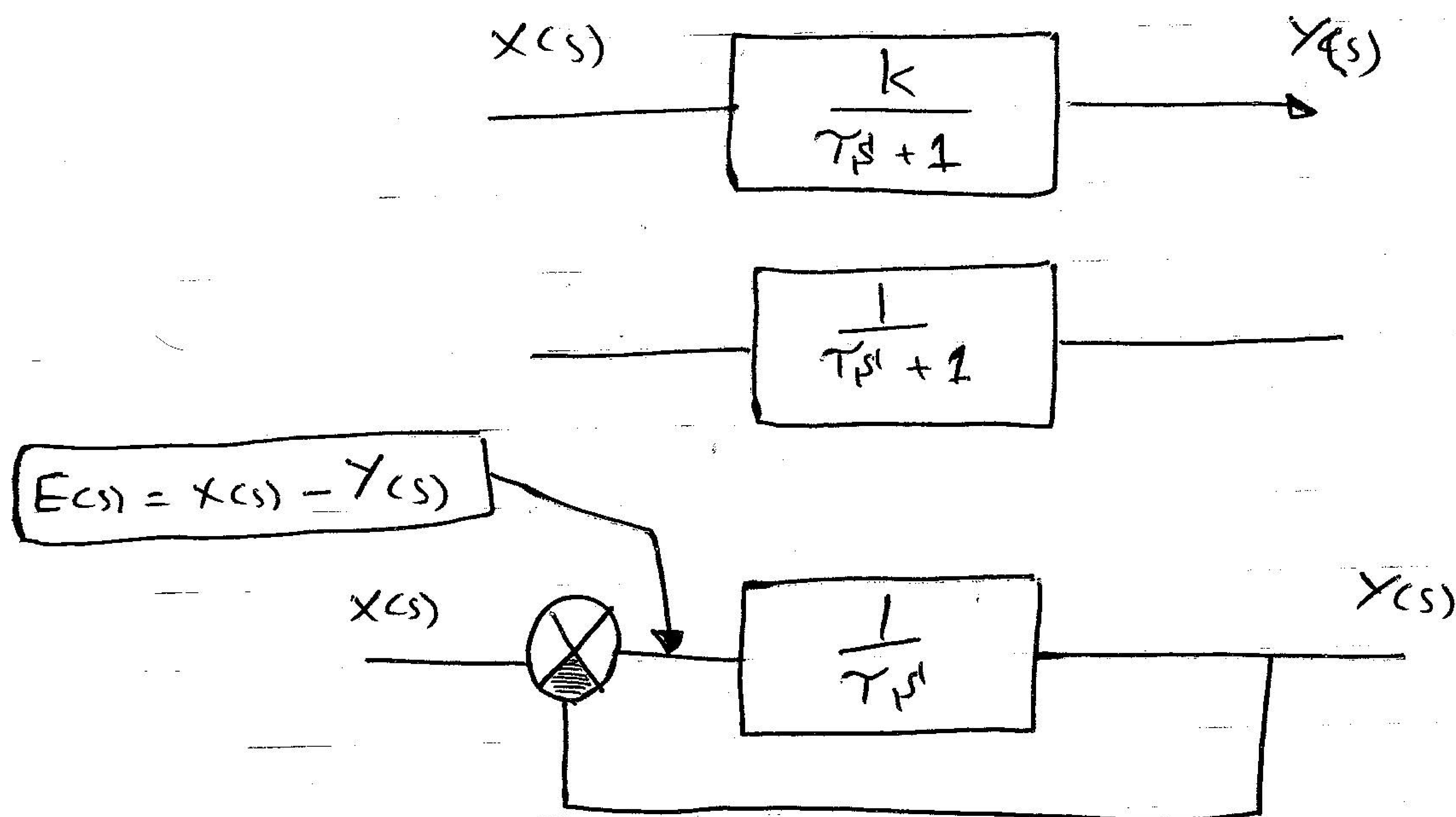
2%] → Criteria
5%]

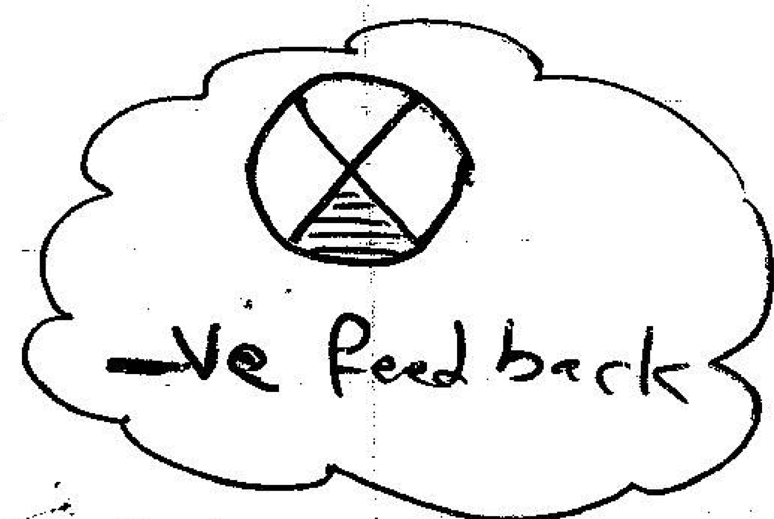
Settling time (T_s) at which the system is said to be stable, and use two criterias 2%, 5%

$$\boxed{T_s \Big|_{5\%} = 3\tau}$$

$$\boxed{T_s \Big|_{2\%} = 4\tau}$$

1^{st} Order System





$e(t) = x(t) - y(t)$

$$e(t)\,\% = \frac{x(t) - y(t)}{x(t)} \times 100\,\%$$

$$y(t)\,\% = \frac{y(t)}{x(t)} \times 100\,\%$$

$e(t) = x(t) - y(t)$
$e(t) = R - kR(1 - e^{-t/\tau})$, $k = 1$
$e(t) = R\,e^{-t/\tau}$

$e(t)\,\% = 1 - y(t)\,\%$

$y(t)\,\% = 1 - e(t)\,\%$

t	0	τ	2τ	3τ	4τ	∞
y%	0%	63%	86%	95%	98%	100%
y	0	$0.63\,y_{ss}$	$0.86\,y_{ss}$	$0.95\,y_{ss}$	$0.98\,y_{ss}$	$y_{ss} = 1$
e%	100%	37%	14%	5%	2%	0%
e	$y_{ss} = 1$	$0.37\,y_{ss}$	$0.14\,y_{ss}$	$0.05\,y_{ss}$	$0.02\,y_{ss}$	0

(*) $e = R e^{-t/\tau}$

$\therefore e\% = \frac{e(t)}{R} \times 100\% = \boxed{e^{-t/\tau}}$

$e(3\tau) = e^{-\frac{3\tau}{\tau}} = e^{-3} = 0.0497 \approx \boxed{0.05}$

$e(4\tau) = e^{-4} = 0.0183 \approx \boxed{0.02}$

$e(2\tau) = e^{-2} \approx \boxed{0.14}$

|
|

and so on --

(*) Finding τ

1. $\boxed{G(s) = \frac{k}{\tau s + (1)}}$ or $\boxed{\tau \dot{y} + y = x(t)}$

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ex $G(s) = \frac{7}{5s + (1)}$ $\therefore k = 7$, $\tau = 5$

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ex $\frac{6}{s+3} \Rightarrow$ divide on 3 $\Rightarrow \frac{\frac{6}{3}}{\frac{s}{3} + 1} = \frac{2}{\frac{1}{3}s + (1)}$

$\therefore k = 2$

$\tau = \frac{1}{3}$

---

ex $y(t) = kR(1 - e^{-t/\tau})$

$y(t) = 7(1 - e^{-5t})$

$k = 7$

$\tau = \frac{1}{5} = \boxed{0.2}$

HW $y(t) = (3 - 2e^{-t/6})$

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ex $2y + 7\dot{y} = 3$ divide on 2

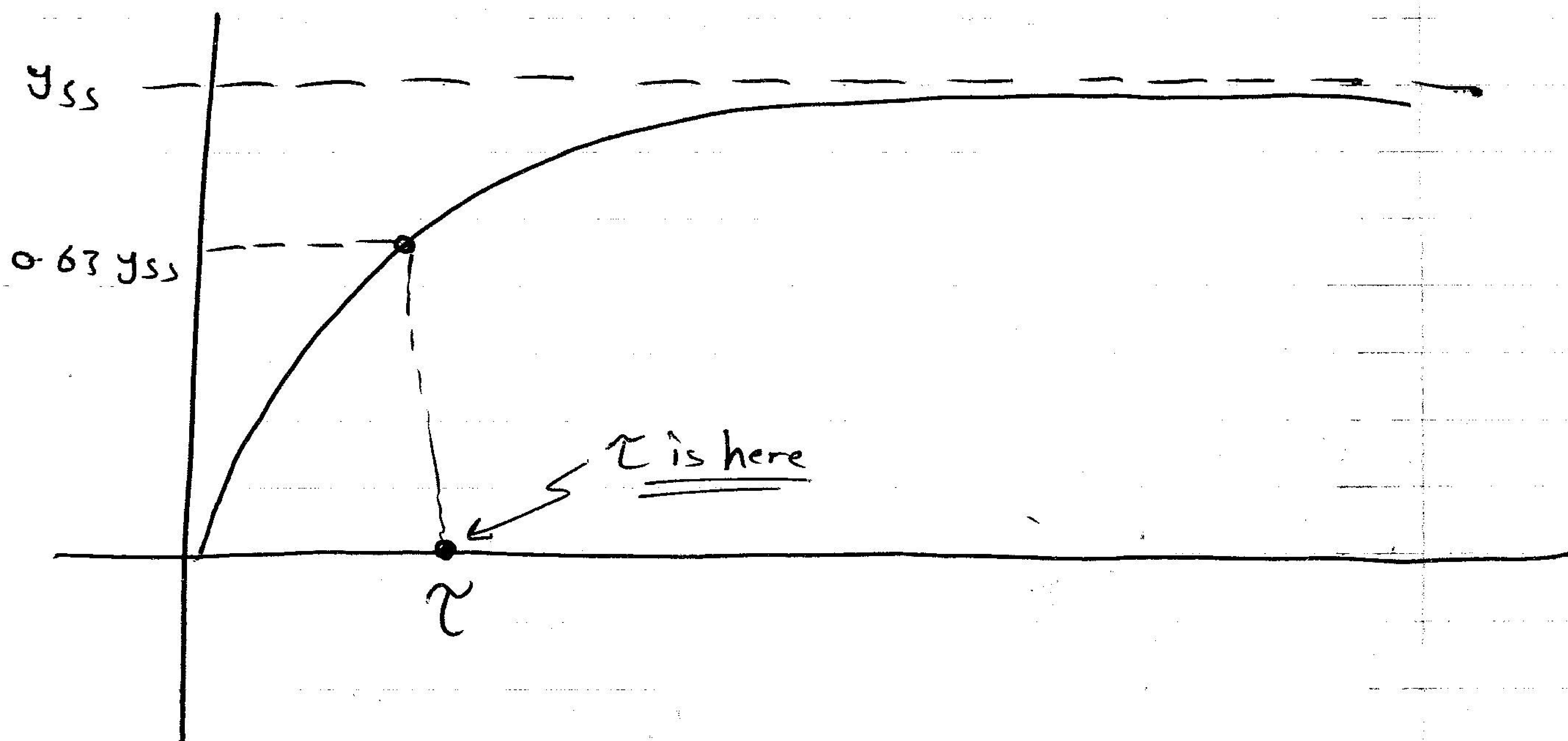
$y + 3.5\dot{y} = \frac{3}{2}$

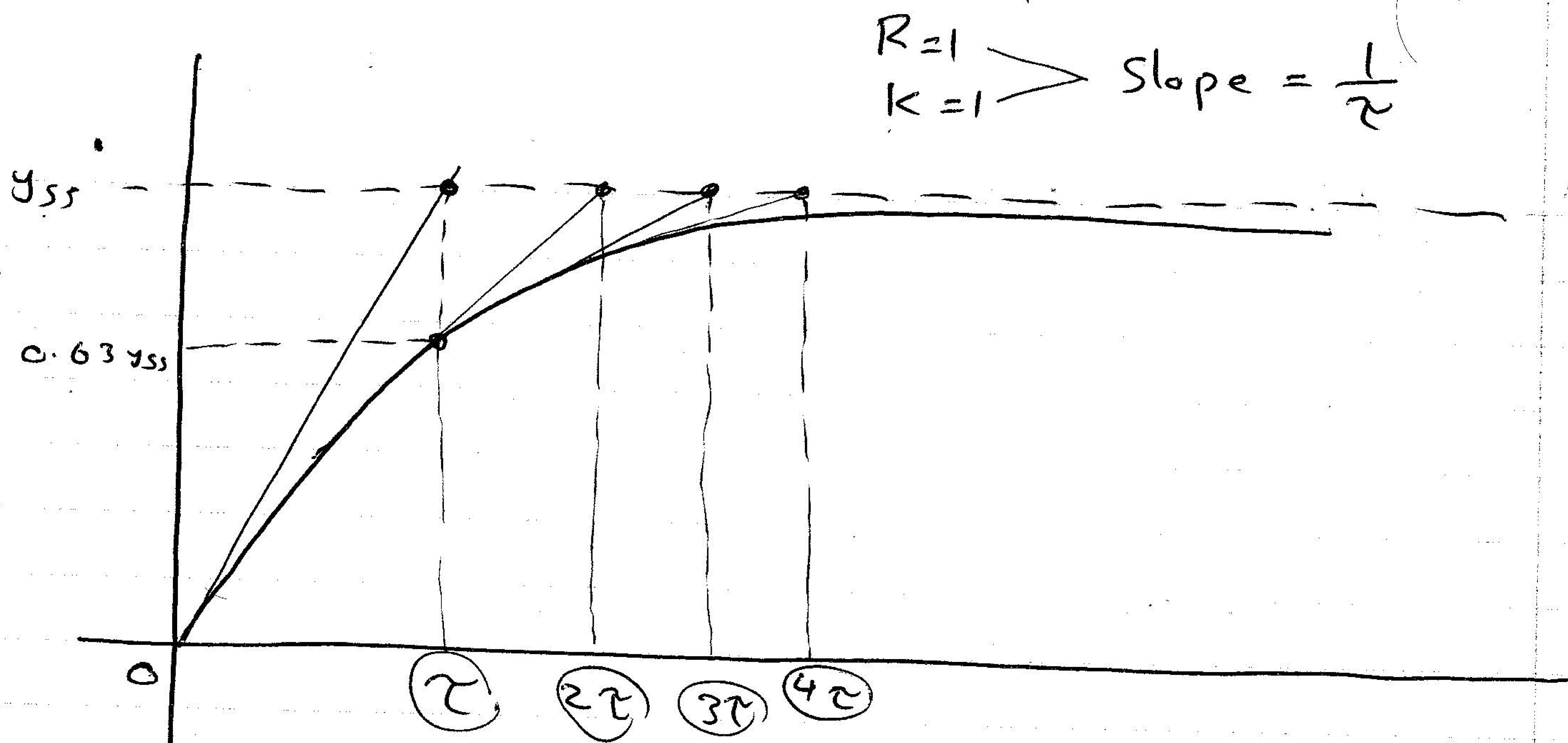
$\therefore \tau = \boxed{3.5}$

$\therefore$ If input $= 1$ $\therefore$ $K = \frac{3}{2}$ not important

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[2] By graph





plot tangent for the system near (0), when the tangent intersect with yss, the corresponding point on the x-axis, will be τ (Slope $= \frac{1}{\tau}$)

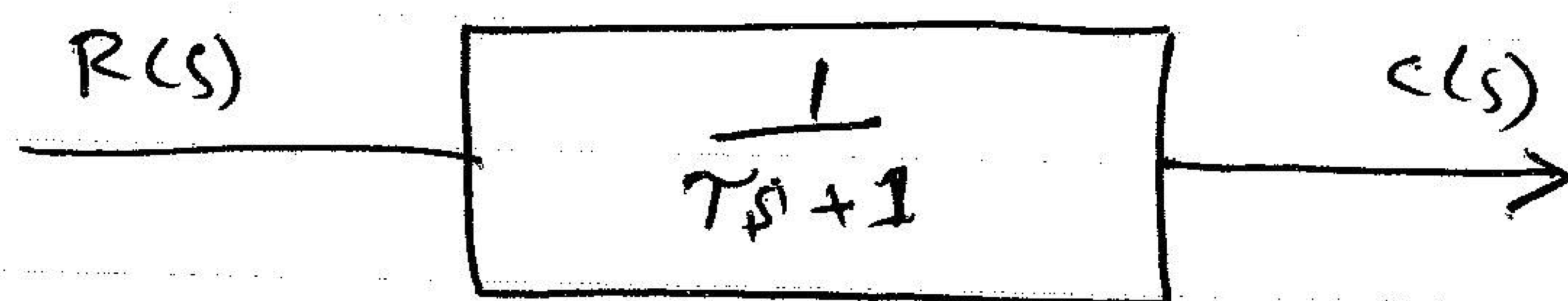
$$y = y_{ss}\,(1 - e^{-t/\tau})$$

$$\dot{y} = y_{ss}\left(\frac{-1}{\tau} * e^{\frac{-t}{\tau}}\right) = \frac{y_{ss}}{\tau}\, e^{\frac{-t}{\tau}} = \boxed{\frac{y_{ss}}{\tau} * t}$$

For ramp input

$r(t) = t$

$R(s) = \frac{1}{s^2}$



$$C(s) = \frac{1}{\tau s+1} * \frac{1}{s^2} = \frac{a}{\tau s+1} + \frac{b}{s^1} + \frac{c}{s^2}$$

$$= \frac{\tau^2}{\tau s + 1} - \frac{\tau}{s} + \frac{1}{s^2}$$

$$\therefore c(t) = t - \tau + \tau e^{-t/\tau}$$

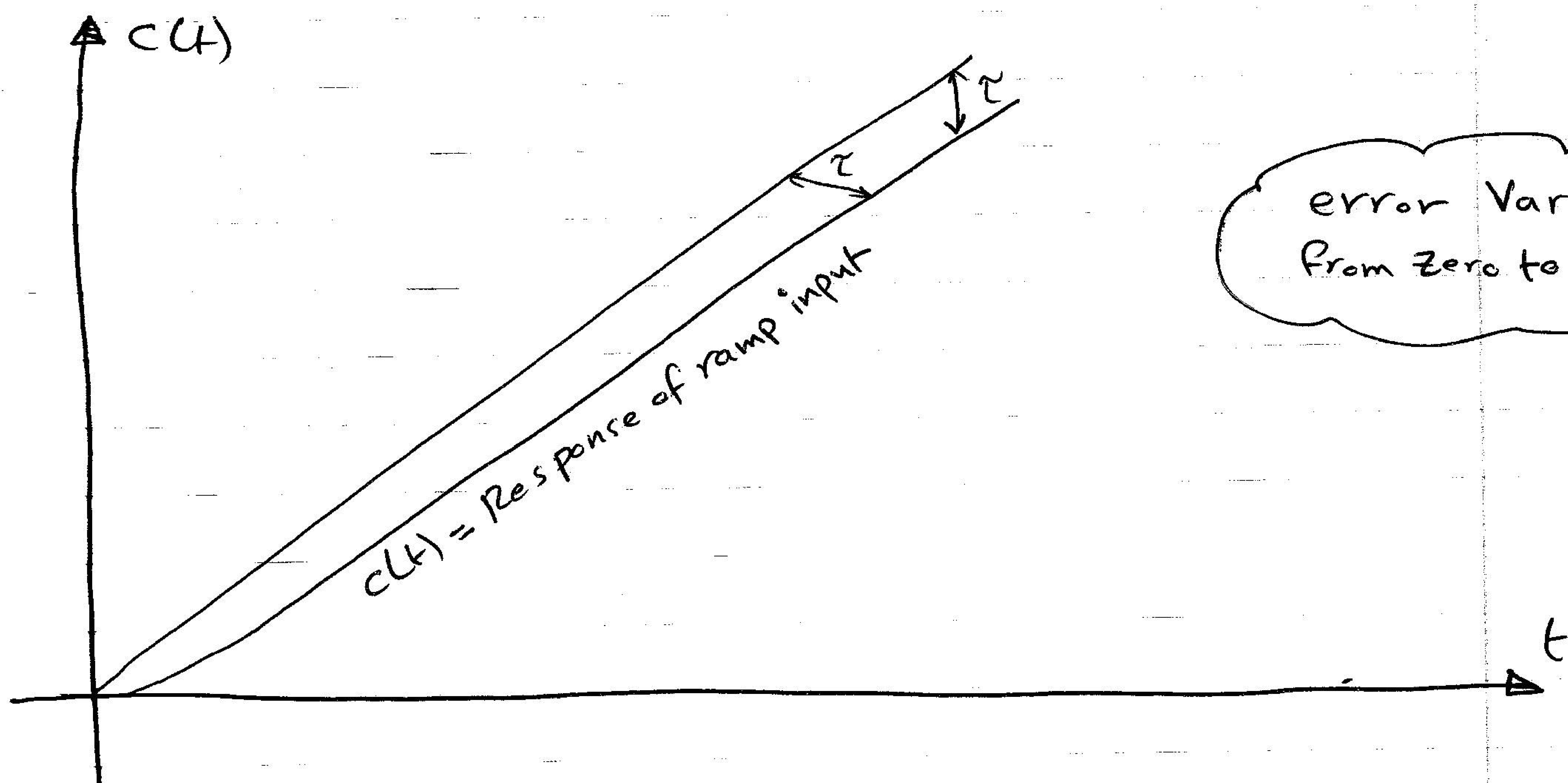
$$e(t) = R(t) - c(t) = t - [t - \tau + \tau e^{-t/\tau}]$$

$$= t - t + \tau - \tau e^{-t/\tau}$$

$$= \tau - \tau e^{-t/\tau}$$

$$\therefore e(t) = \tau[1 - e^{-t/\tau}]$$

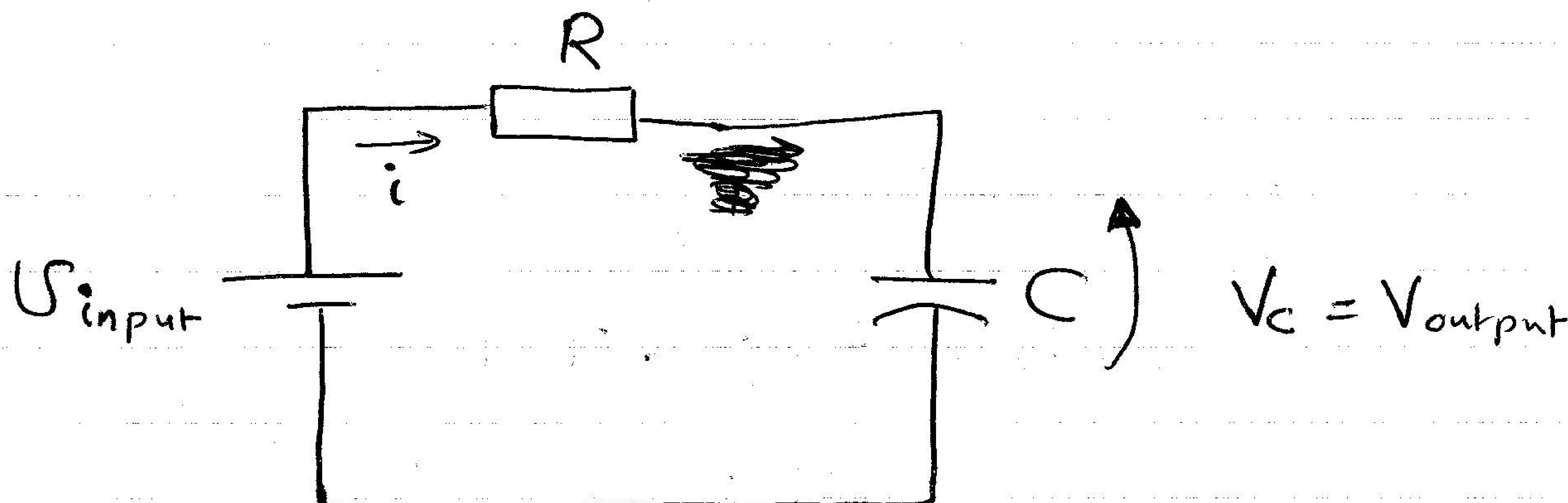
$$e_{ss} = \tau$$



error Varies from zero to τ

Variable = small letter (i)
Constant = Capital letter (I)

Ex1: Charging Capacitor



$V_i = V_R + V_c$

$V_i = IR + \frac{1}{C}\int i\,dt$

$V_o = \frac{1}{C}\int i\,dt$

$\dot{V}_o = \frac{i}{C} \Rightarrow i = C\dot{V}_o$

$V_i = RC\dot{V}_o + V_o$

$IR = C\dot{V}_o R$

but standard equation = $\tau\dot{y} + y = KX$

$\rightarrow \tau\dot{y} + y = KX$

$\rightarrow RC\dot{V}_o + V_o = V_i$

$\therefore \tau = RC$

$K = 1$

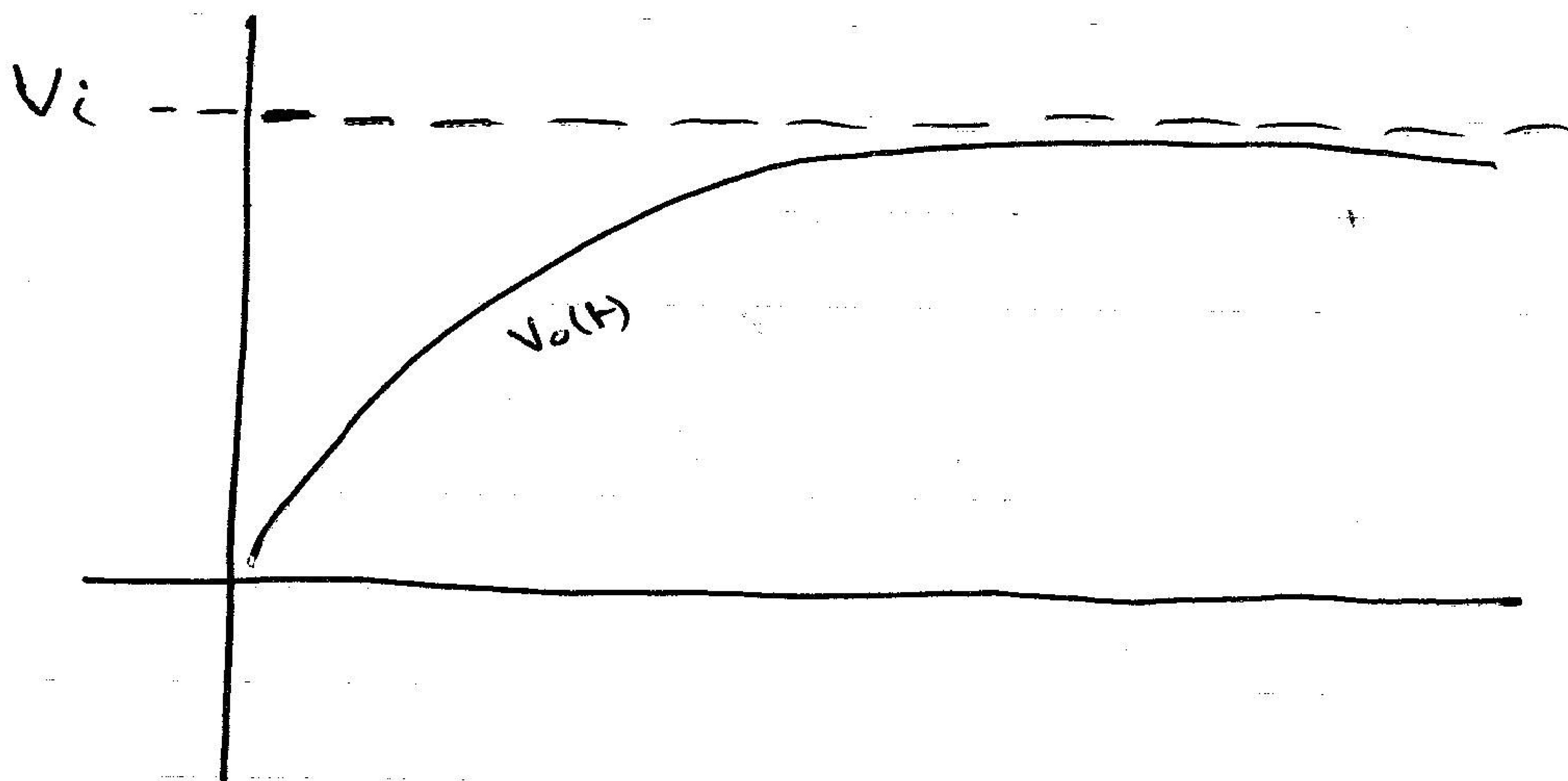
$V_I(s) = RCS\,V_o(s) + V_o(s)$

$V_I(s) = V_o(s)\,[RCS + 1]$

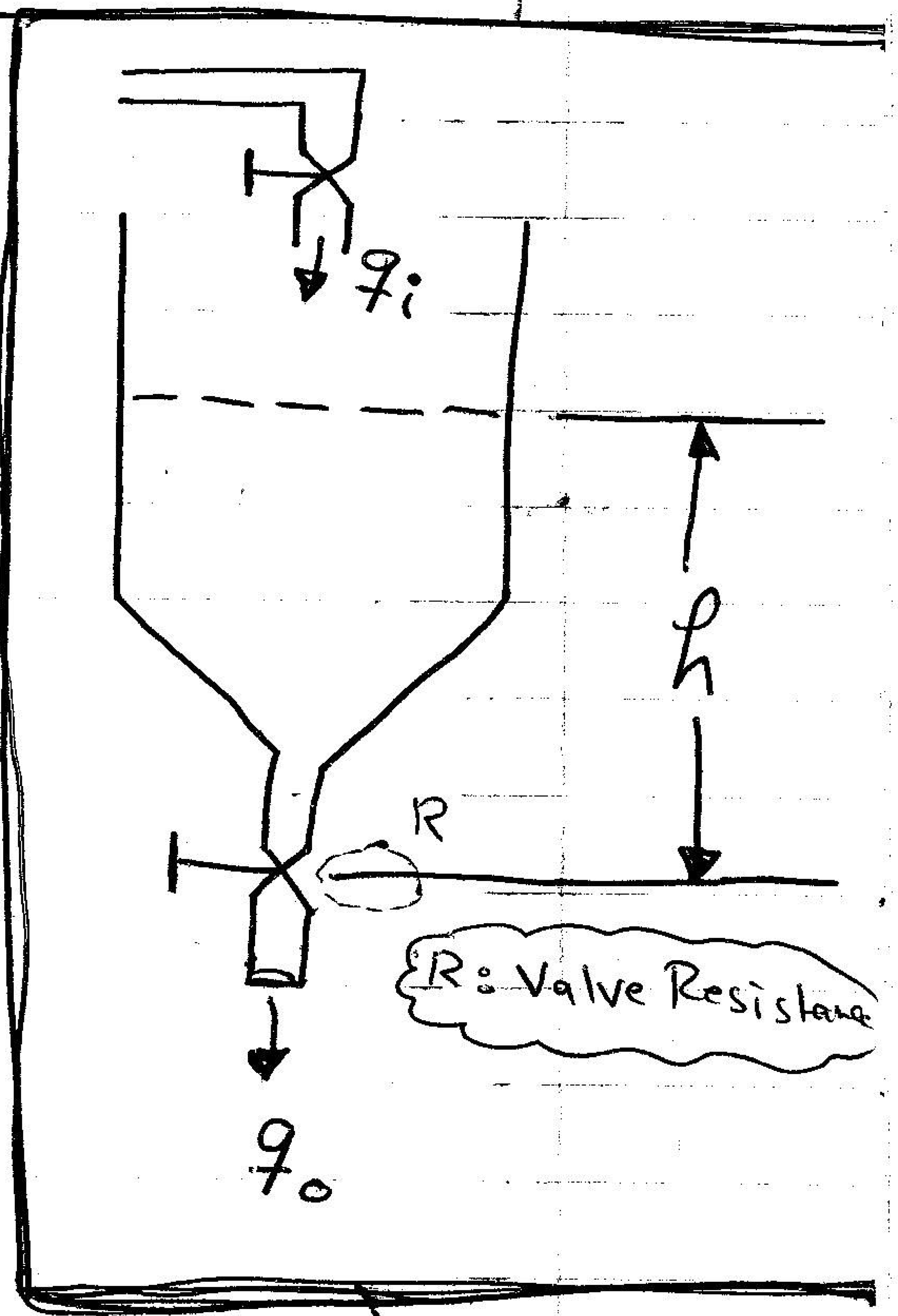
$$\frac{V_o(s)}{V_I(s)} = \frac{1}{RCs+1}$$

⇒ ⊛ for step input

$$V_o(t) = V_i\left(1 - e^{-t/RC}\right)$$



Ex 2 : Tank filling



Volumetric flow rate = $q_i - q_o$ (Volume/s) وحدته

$$h = \frac{1}{A}\int q_i - q_o \cdot dt \Rightarrow$$

$$\dot{h} = \frac{1}{A}(q_i - q_o)$$

but $q_o = \frac{P}{R} = \frac{\rho g h}{R}$

$$\therefore \frac{1}{A}\left(q_i - \frac{\rho g}{R} h\right) = \dot{h}$$

Standard equation
$\tau \dot{y} + y = kx$

$$\frac{1}{A} q_i - \frac{\rho g}{RA} h = \dot{h}$$

$$\left(\dot{h} + \frac{\rho g}{RA} h = \frac{1}{A} q_i\right) \times \frac{RA}{\rho g}$$

$$\frac{RA}{\rho g} \dot{h} + h = \frac{R}{\rho g} q_i$$

$$\updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow \quad \updownarrow$$

$$\tau \dot{y} + y = k x$$

$$\therefore \tau = \frac{RA}{\rho g}$$

$$k = \frac{R}{\rho g}$$

standard

$$\therefore \frac{\text{output}}{\text{Input}} = \frac{h}{q_i} = \frac{k}{\tau s + 1} = \frac{\frac{R}{\rho g}}{\frac{RA}{\rho g} s + 1}$$

$$\therefore h(t) = \frac{R}{\rho g} q_i \left(1 - e^{-\frac{\rho g}{RA} t}\right)$$

2nd Order Systems

2nd order system : is the system that its output rates with the change of its derivative.

and can be expressed by the following derivative equation:

[redacted]

$$\ddot{y} + 2\zeta \omega_n \dot{y} + \omega_n^2 y = k \omega_n^2 X$$

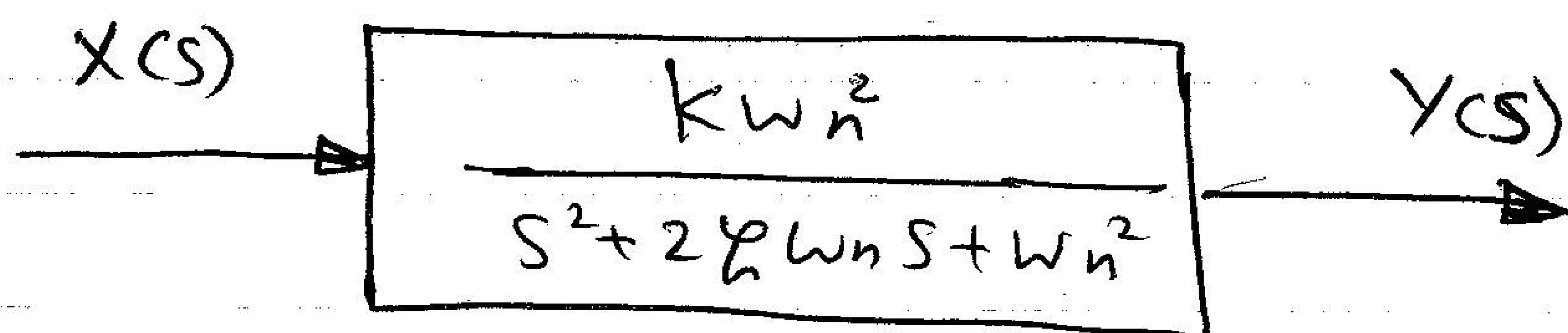
- ⊛ k : DC gain.
- ⊛ $\omega_n = \omega_o$: natural frequency.
- ⊛ ζ : damping ratio
- ⊛ $a = 2\zeta$: damping factor

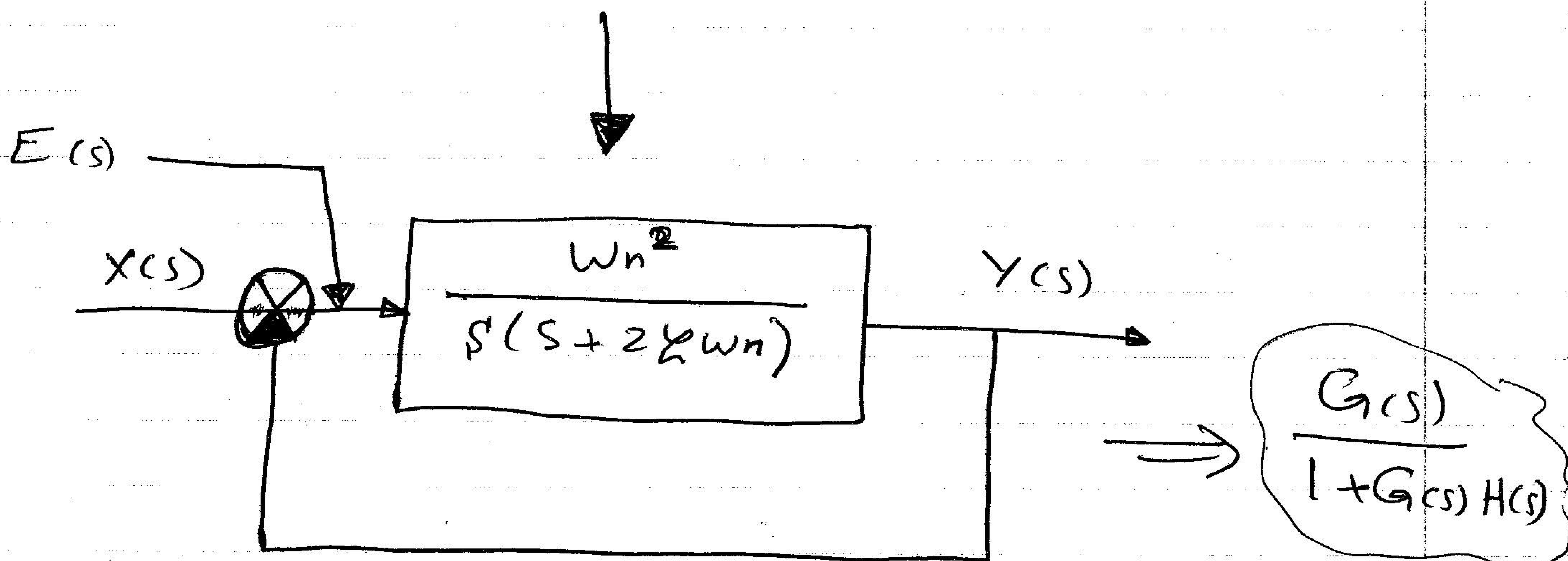
ω = omega
ζ = Zeta

$$S^2 Y(s) + 2\zeta \omega_n S\, Y(s) + \omega_n^2 Y(s) = k \omega_n^2 X(s)$$

$$Y(s)\left[S^2 + 2\zeta \omega_n S + \omega_n^2\right] = k \omega_n^2 X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{k \omega_n^2 \text{[redacted]}}{S^2 + 2\zeta \omega_n s + \omega_n^2}$$





$$\frac{\frac{\omega_n^2}{S(S+2\zeta\omega_n)}}{1+\frac{\omega_n^2}{S(S+2\zeta\omega_n)}} * \frac{S(S+2\zeta\omega_n)}{S(S+2\zeta\omega_n)}$$

$$= \frac{\omega_n^2}{S(S+2\zeta\omega_n)+\omega_n^2} = \boxed{\frac{\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}}$$

$X(t) =$ Unit step input $= \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$

$$Y(s) = \frac{K\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2} X(S)$$

$\Rightarrow$ for $X(S) = \frac{1}{S}$

$$\Rightarrow Y(s) = \frac{K w_n^2}{s^2 + 2\zeta w_n s + w_n^2} * \frac{1}{s}$$

CASE 1 $\boxed{\zeta > 1}$ The system is called **OVER DAMPED**

There is two poles for $\frac{Y(s)}{X(s)}$, the poles are <u>-ve</u> and <u>real</u>

$$S_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(هنا رابثت ان الجذرين negative Real)

$$= \frac{-2\zeta w_n \pm \sqrt{(2\zeta w_n)^2 - 4 w_n^2}}{2}$$

$$= -\zeta w_n \pm \sqrt{\zeta^2 w_n^2 - w_n^2}$$

$$= -\zeta w_n \pm w_n \sqrt{\zeta^2 - 1}$$

(*) Real: because $\boxed{\zeta > 1} \Rightarrow \therefore \boxed{\sqrt{\zeta^2 - 1} > 1}$ $\therefore$ (No imaginary parts)

$\zeta w_n \square w_n \sqrt{\zeta^2 - 1}$

$\zeta^2 w_n^2 \boxed{>} w_n^2 \zeta^2 - w_n^2$

(*) $\therefore$ negative

$$\Rightarrow y(t) = 1 + \frac{w_n}{2\sqrt{\zeta^2 - 1}} \left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2} \right)$$

(ليس للحفظ)

(time domain Response)

In Case S_2 is too Small and very close to Zero :-

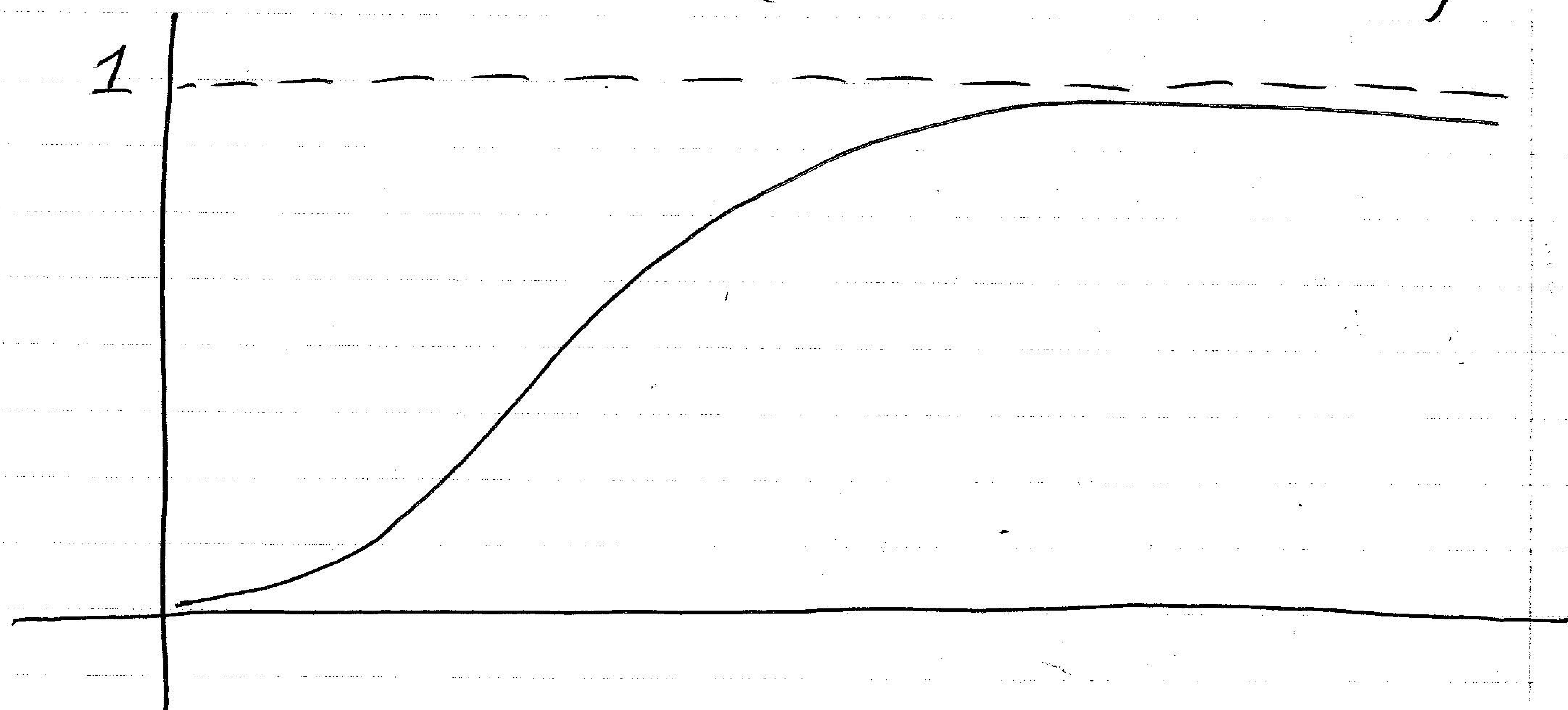
* $C(t)$ Can be written as :

$$C(t) = 1 - e^{-(\zeta - \sqrt{\zeta^2 - 1})W_n t}$$

* for input $X(s) = R$ & Gain = K

$$C_n(t) = KRC(t)$$

$$= KR\left(1 - e^{-(\zeta - \sqrt{\zeta^2-1})W_n t}\right)$$



CASE 2 & CASE 3 will be discussed after EID AL-ADHA

Regards

Sun; 20/10/2013

CASE 2 Critically damped Oscillation ($\zeta = 1$)

$$S_{1,2} = \left(\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n \Rightarrow \boxed{\zeta = 1}$$

$$S_{1,2} = -\omega_n$$

$$C(s) = \frac{\omega_n^2}{(s+\omega_n)^2\, s} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} * \frac{1}{s}$$

$$\mathcal{L}^{-1} C(s) = \frac{\omega_n^2}{(s+\omega_n)^2\, s} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} * \frac{1}{s}$$

~~$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)$~~

~~$\theta = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$~~

$$\Rightarrow \boxed{c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)} \Rightarrow \text{CASE 2 equation}$$

CASE 3 Underdamped Oscillation

$$\zeta < 1$$

There are two-pole Conjugate

$$C(t) = 1 - \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \sin(w_d t + \theta)$$

$$\theta = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$\sin t + \cos t = \sqrt{2} \sin(t + \pi/4)$

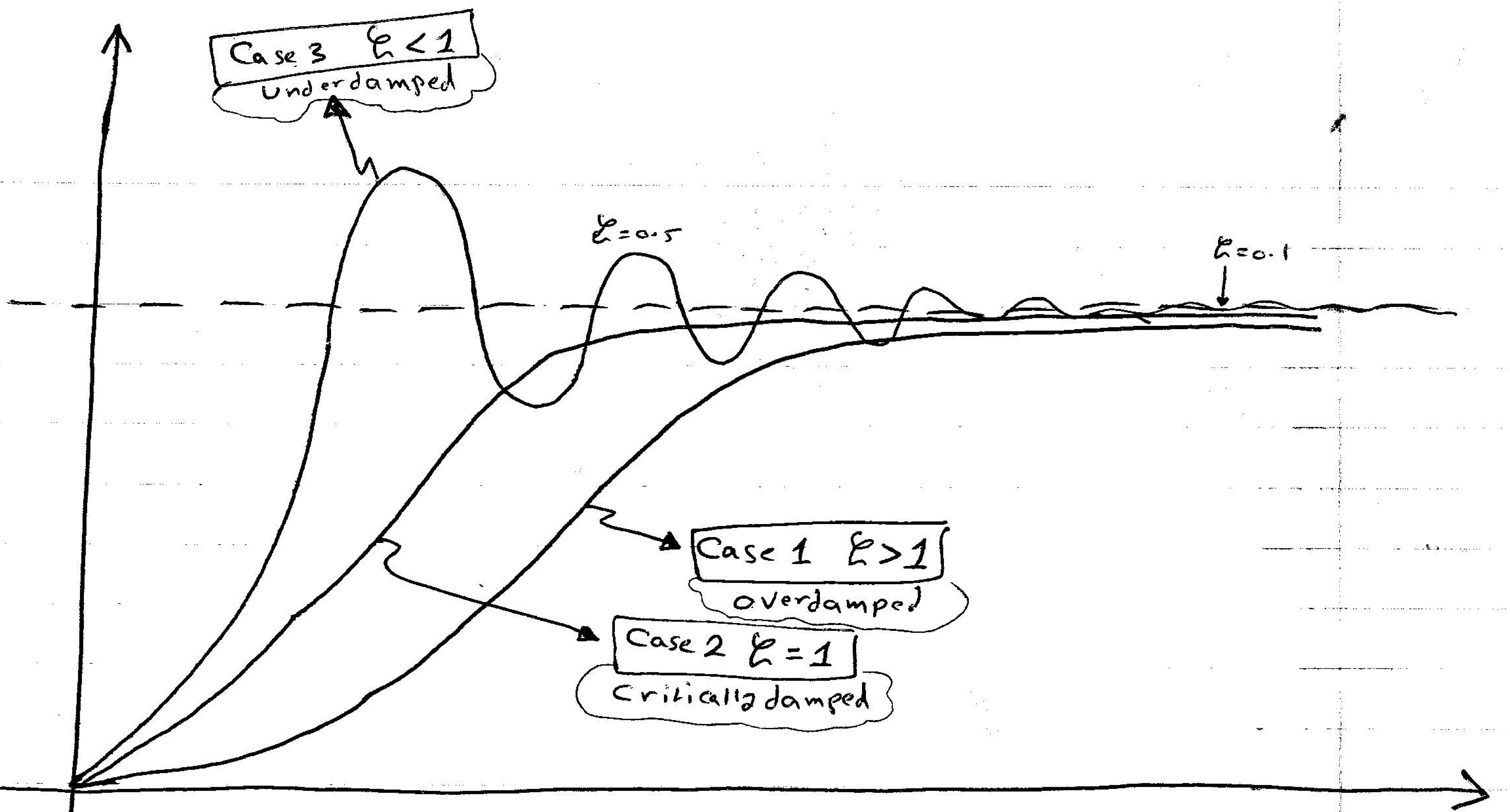
Another form of $C(t)$ [less used] (أقل استخداماً)

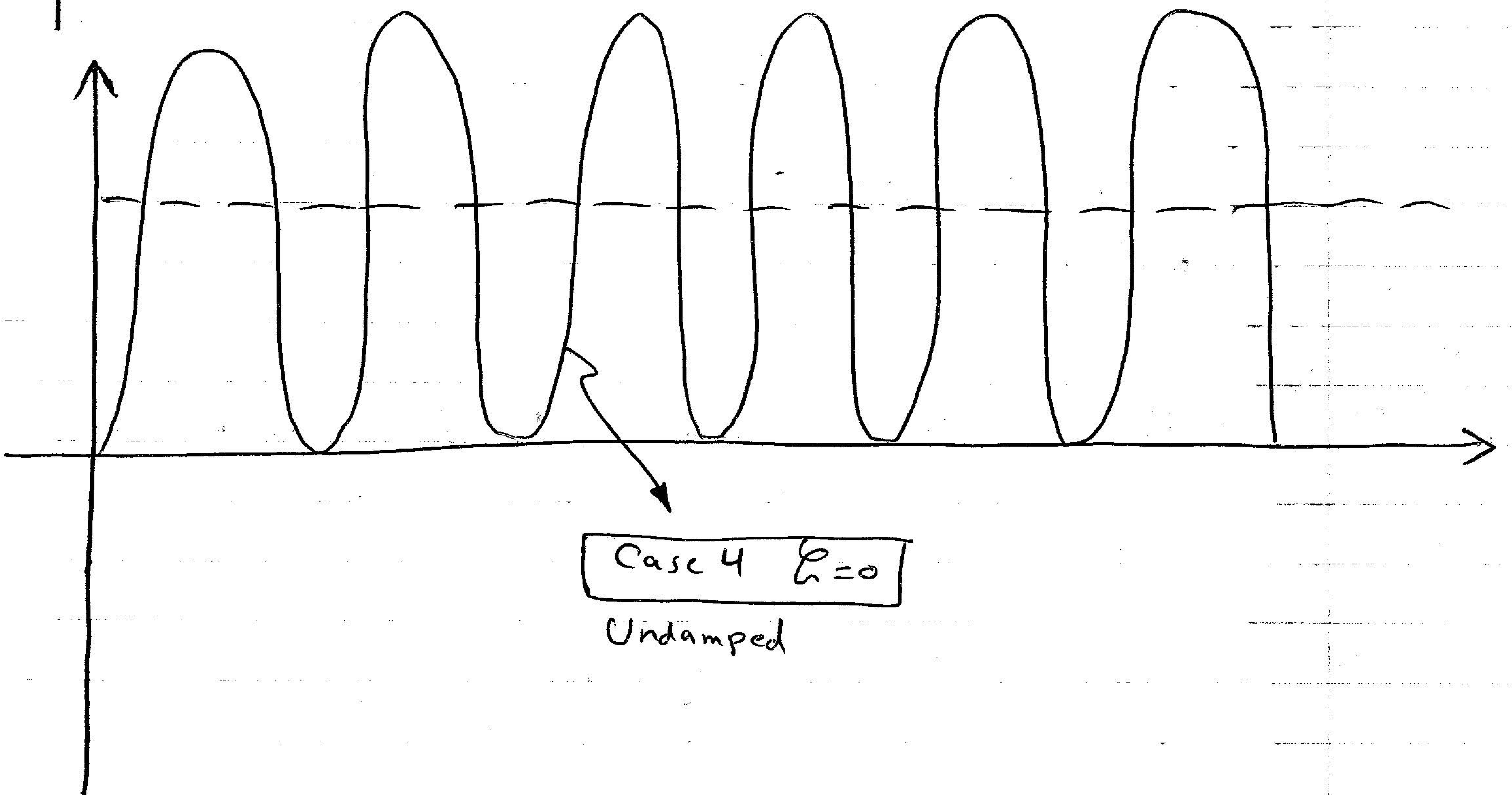
$$C(t) = 1 - e^{-\zeta w_n t}\left[\cos w_d t + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin w_d t\right]$$

CASE 4 Undamped Oscillation $\zeta = 0$

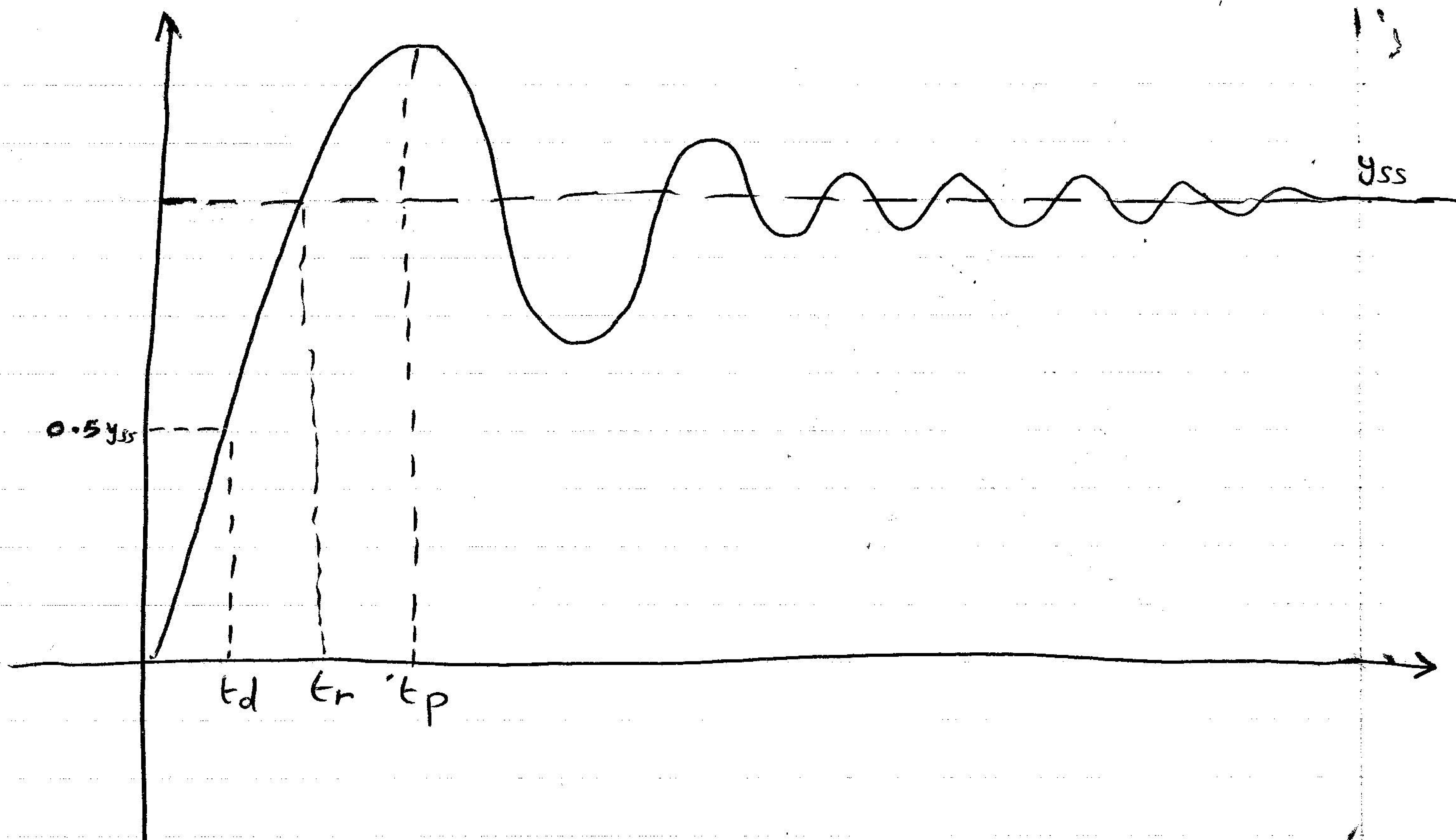
$$C(t) = 1 - \cos w_n t$$

(*) $$w_d = w_n \sqrt{1-\zeta^2}$$


Case 3 $\zeta < 1$
Underdamped
$\zeta = 0.5$
$\zeta = 0.1$
Case 1 $\zeta > 1$
overdamped
Case 2 $\zeta = 1$
Critically damped


Case 4 $\zeta = 0$
Undamped

Tue; 22/10/2013

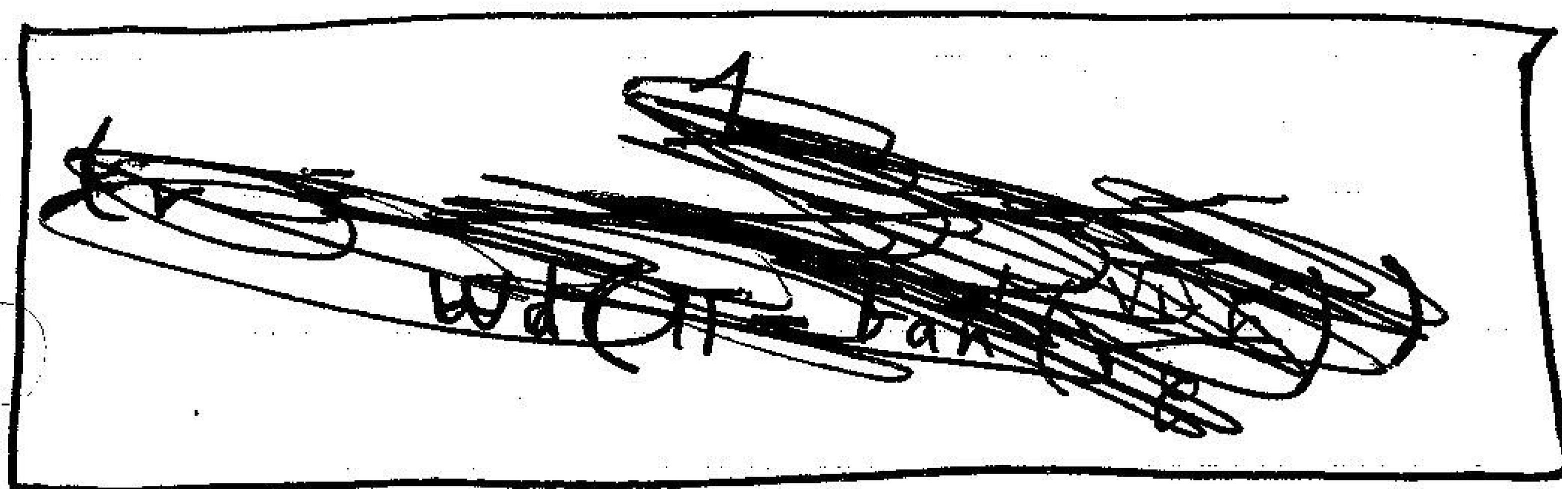


1. y_{ss} : is the final value at (∞) reached by the system $= KR$

2. Delay time : is the time required by the system to reach 50% of the final value. (t_d)

3. Rising time (t_r) : is the time taken by the signal to reach
   [0 → 100%] for underdamped Response.
   [5 → 95%] for Critically damped Response.
   [10 → 90%] for overdamped Response.

for underdamped :



$$t_r = \frac{1}{w_d}\left(\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

4. Peak time (t_p) : is the time required for the response to reach the first peak

by taking the 1st derivative of response equation :

$$\frac{dc(t)}{dt} = 0$$

$$[\sin \omega_d t_p] \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t_p} = 0$$

$[\sin \omega_d t_p]$ → Can be Zero!

$\frac{\omega_n}{\sqrt{1-\zeta^2}}$ → impossible to be zero

$e^{-\zeta \omega_n t_p}$ → zero when $t_p = \infty$ X

$\therefore \Rightarrow \sin \omega_d t_p = 0$

$\therefore \omega_d t_p = 0, \boxed{\pi}, 2\pi, 3\pi, 4\pi, ——$

$\therefore \omega_d t_p = \pi$

$$\boxed{\therefore t_p = \frac{\pi}{\omega_d}}$$ 1st Peak time

$$\boxed{t_{p_2} = \frac{3\pi}{\omega_d}}$$ 2nd Peak time ,

and So on — ⟹

⊛ Remember : $\boxed{\omega_d = \omega_n \sqrt{1-\zeta^2}}$

5. y_p = maximum response

$$y(t_p) = 1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

6. Maximum overshoot M_p

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = y_p - 1$$

7. Overshoot Percentage

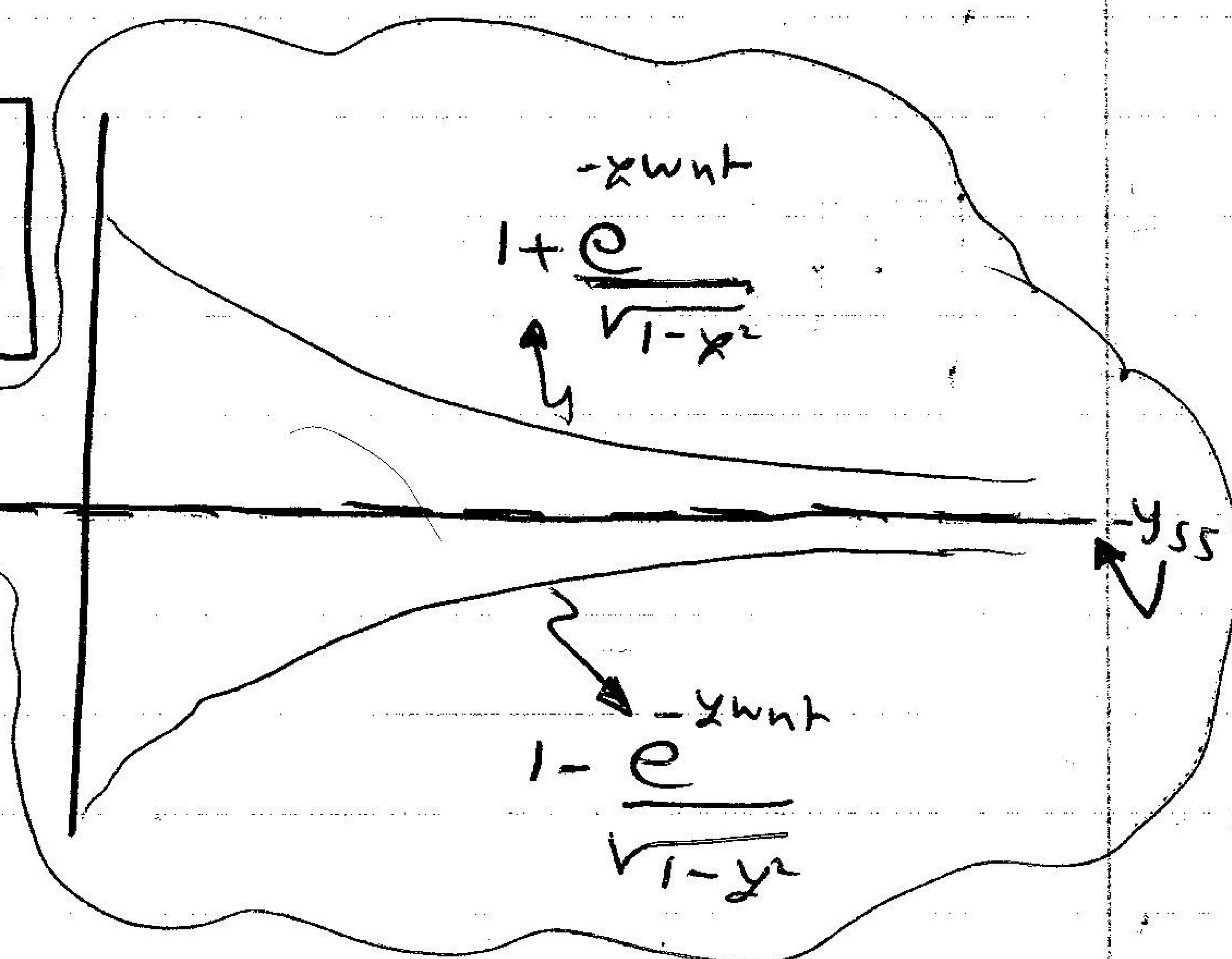
$$M_p\% = \frac{M_p}{y_{ss}} * 100\% = e^{-\zeta\pi/\sqrt{1-\zeta^2}} * 100\%$$

in the case ($y_{ss} = 1$)

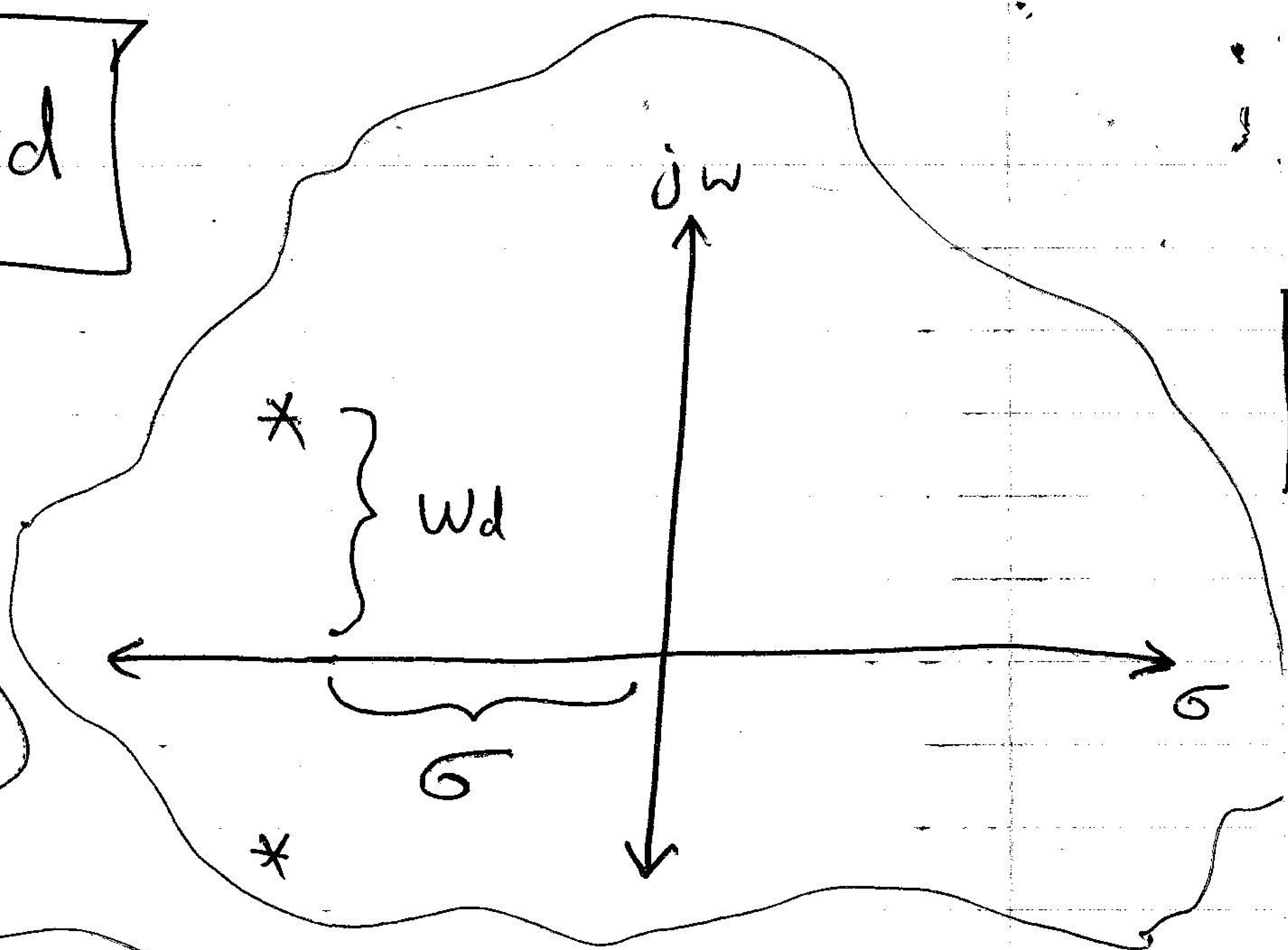
8. Settling time (t_s)

$$t_s\Big|_{5\%} = 3\tau = \frac{3}{\sigma} = \frac{3}{\zeta\omega_n}$$

$$t_s\Big|_{2\%} = 4\tau = \frac{4}{\sigma} = \frac{4}{\zeta\omega_n}$$



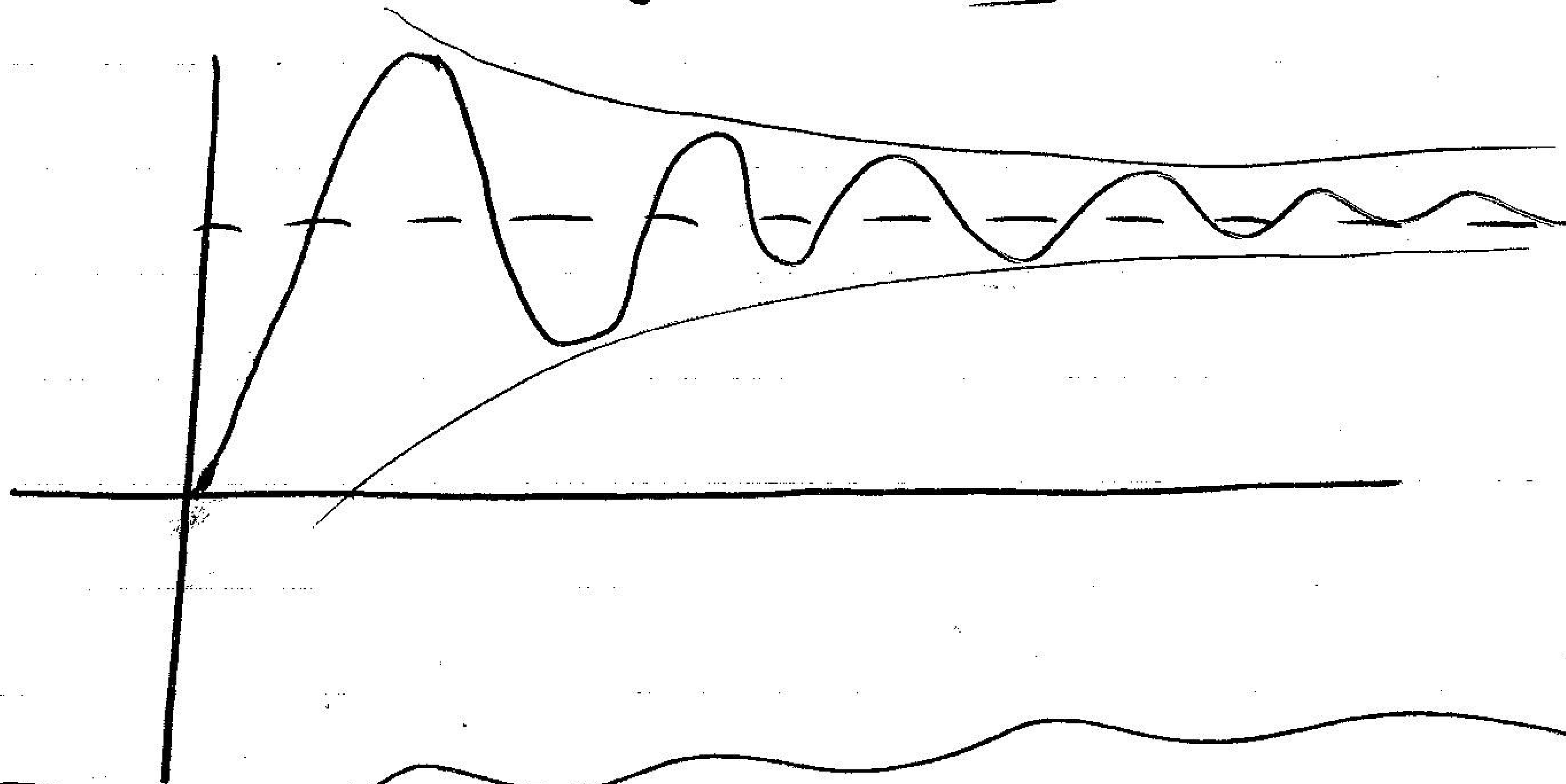
$$S = \sigma + jw_d$$



- σ goes bigger
  ⇒ the system is fasten to be stable
- Wd goes bigger
  ⇒ the frequency of the system will be larger

9. Exponential Decay ratio

is measured by dividing the 2nd peak by the 1st peak

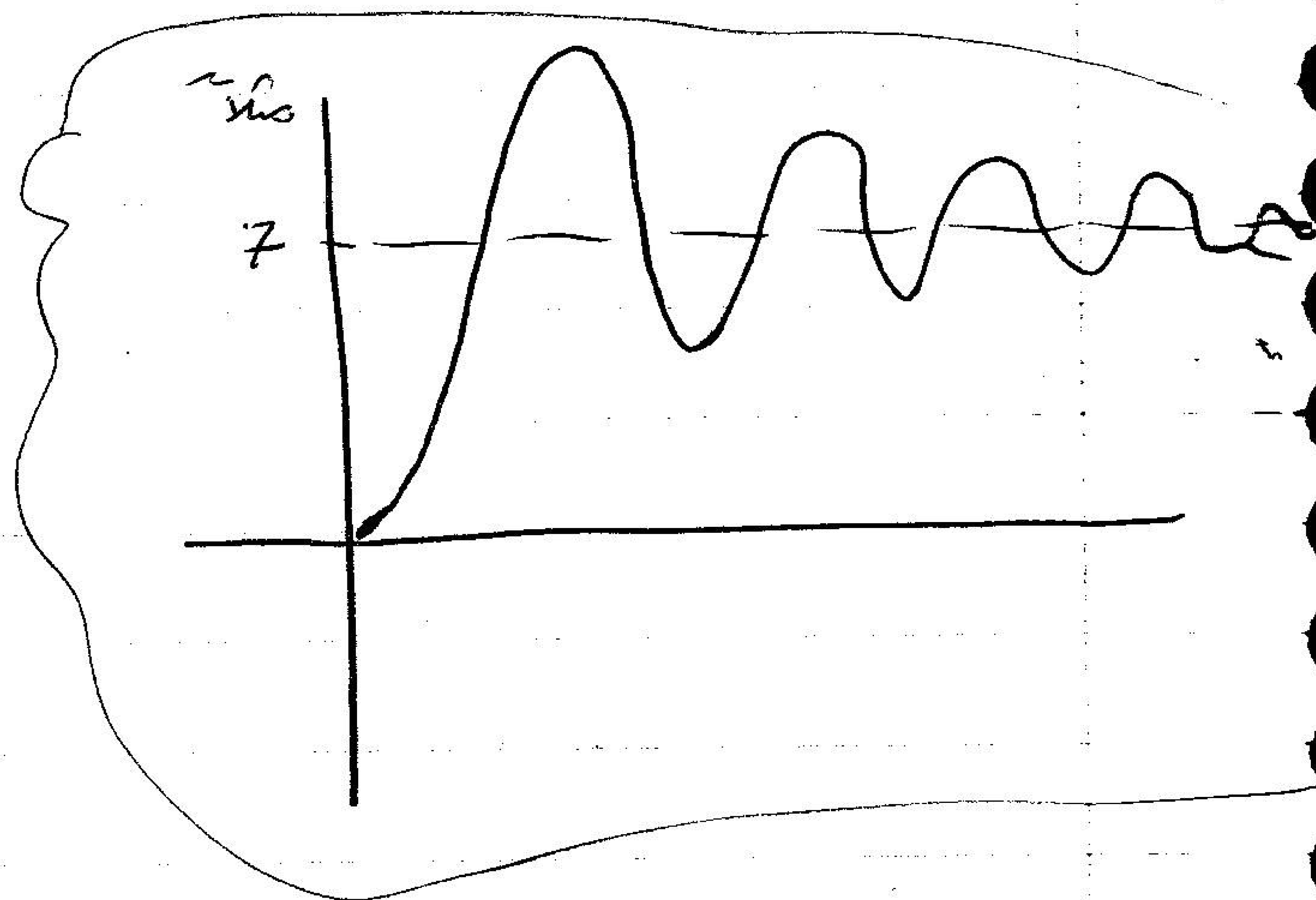


* exponential decay ratio $= \frac{M_{P2}}{M_{P1}} = \frac{M_{P3}}{M_{P2}}$

∴ exponential decay ratio $= \frac{M_{P2}}{M_{P1}} = \frac{e^{-3\zeta\pi/\sqrt{1-\zeta^2}}}{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$

$$= \boxed{e^{-2\zeta\pi/\sqrt{1-\zeta^2}}}$$

Ex Find ζ if the E.D.R = 50 %



$$E.D.R = 0.5 = e^{-2\zeta\pi/\sqrt{1-\zeta^2}}$$

$$\therefore \ln 0.5 = \frac{-2\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\therefore -0.7 = \frac{-2\zeta\pi}{\sqrt{1-\zeta^2}} \Rightarrow 0.7 = \frac{2\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow 0.7\sqrt{1-\zeta^2} = 2\pi\zeta$$

$$\Rightarrow 0.7\sqrt{1-\zeta^2} = 6.28\,\zeta$$

$$\Rightarrow \sqrt{1-\zeta^2} = 9\zeta \Rightarrow 1-\zeta^2 = 81\zeta^2$$

$$\Rightarrow 82\zeta^2 = 1 \Rightarrow \zeta = \sqrt{\frac{1}{82}} = \boxed{0.11}$$

Example:- For a 2nd order system, $\zeta = 0.6$, $w_n = 5$ rad/sec, subjected to a unit step input.

Find :-

① w_d ② σ ③ t_r ④ t_p ⑤ M_p ⑥ t_s

⑦ E.D.R . then draw the response.

Solution :-

① $w_d = w_n\sqrt{1-\zeta^2} = 5\sqrt{1-0.36} = 5\sqrt{0.64} = 5(0.8) = \boxed{4}$

② $\sigma = \zeta w_n = 0.6(5) = \boxed{3}$

③ $t_r = \frac{\pi - \beta}{w_d}$ $\left[\beta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = \tan^{-1}\frac{w_d}{\sigma}\right]$

$\beta = \tan^{-1}\frac{w_d}{\sigma} = \tan^{-1}\frac{4}{3} = 53^\circ = \boxed{0.92 \text{ radian}}$

$\therefore t_r = \frac{\pi - 0.92}{4} = \boxed{0.55 \, s}$

④ $t_p = \frac{\pi}{w_d} = \frac{\pi}{4} = \boxed{0.78 \, s}$

⑤ $M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = e^{-\pi\frac{\sigma}{w_d}} = e^{-\frac{3}{4}\pi} = \boxed{0.1}$

$M_p\% = 0.1 \times 100\% = \boxed{10\%}$

⑥ $t_s|_{5\%} = \frac{3}{\sigma} = \frac{3}{3} = \boxed{1 \, s}$

$t_s|_{2\%} = \frac{4}{\sigma} = \frac{4}{3} = \boxed{1.3 \, s}$

⑦ $E.D.R = e^{-2\zeta\pi/\sqrt{1-\zeta^2}} = [M_{p_1}]^2$ ~~0.1~~ ~~0.01~~

$= [0.1]^2 = \boxed{0.01}$

⊛ Drawing the response :-

$e < 5\%$ — $e < 2\%$

1.1 — 0.1 — 0.09 — 0.08 — 0.07

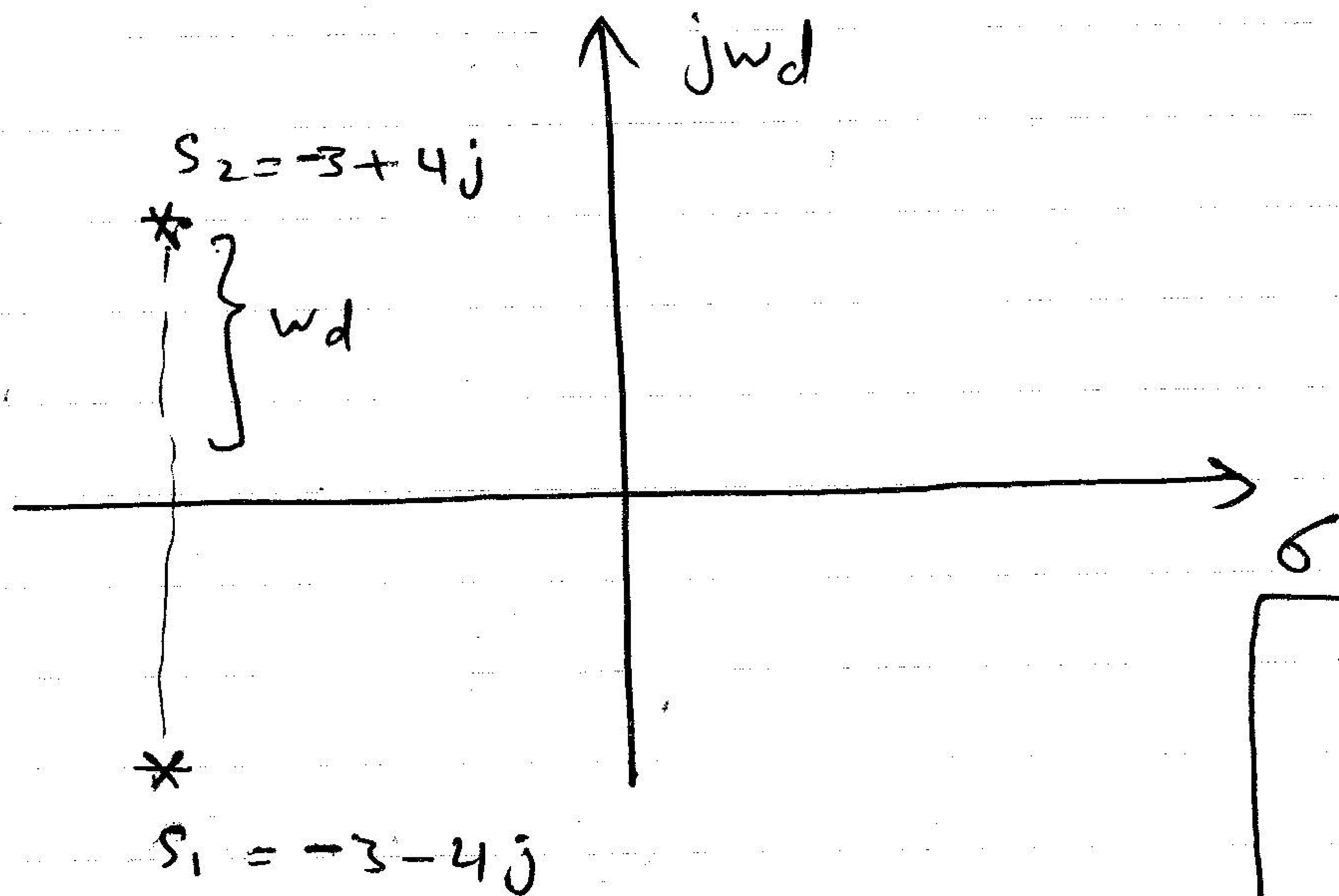
$R(t) = 1$ — $y_{ss} = 1$

0.55 → t_r — 0.78 → t_p — $t_s = 1$

$t_r \leftrightarrow t_p$: β

β فرق الزاوية

⊛ Roots of the system $S = \sigma + j\omega_d$



This system is stable
Roots are −ve

Roots location (2 roots)

⊛ G(s) for the system $= \dfrac{K\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2} = \boxed{\dfrac{25}{S^2 + 6S + 25}}$ T.F.

Ex (H.W # 1)

Find W_n & ζ

So that the PO% = 25%

and Settling time at 5% Criteria = 3 s

Find the other specs.

$$\circledast \; e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.25 \Rightarrow \ln e^{-\zeta\pi/\sqrt{1-\zeta^2}} = \ln 0.25$$

$$\Rightarrow \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = -1.386 \Rightarrow \frac{+3.14\zeta}{\sqrt{1-\zeta^2}} = +1.386$$

$$\Rightarrow \frac{9.869\zeta^2}{1-\zeta^2} = 1.921 \Rightarrow 9.869\zeta^2 = 1.921 - 1.921\zeta^2$$

$$11.79\zeta^2 = 1.921 \Rightarrow \zeta^2 = \frac{1.921}{11.79} \approx 0.16 \Rightarrow \boxed{\zeta = 0.4}$$

$$t_s\Big|_{5\%} = \frac{3}{\zeta W_n} = 3 \Rightarrow \therefore \zeta W_n = 1$$

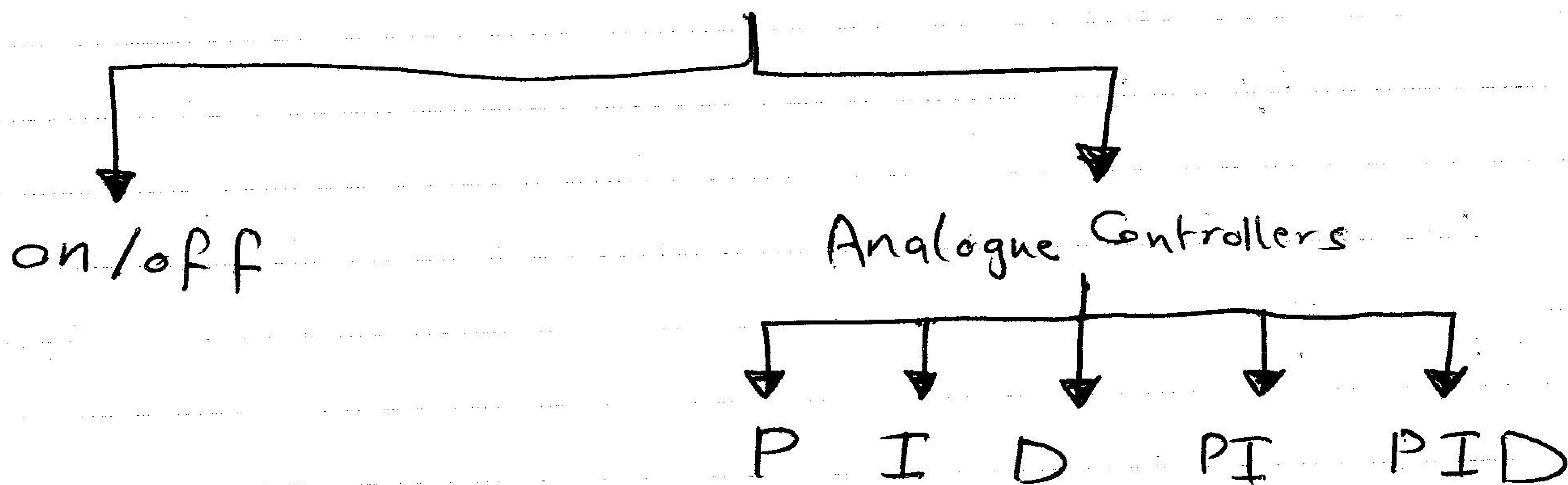
$$\Rightarrow 0.4 W_n = 1 \Rightarrow \boxed{W_n} = \frac{1}{0.4} = \boxed{2.5}$$

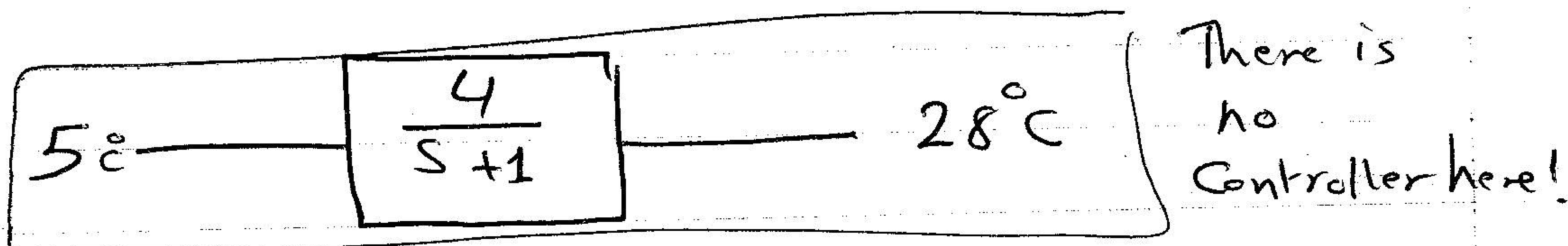
$\zeta = 0.4$
$W_n = 2.5$

We find ζ and W_n $\Rightarrow \therefore$ now we can find all the other Specs :-)

شهر ١٠/٢٩

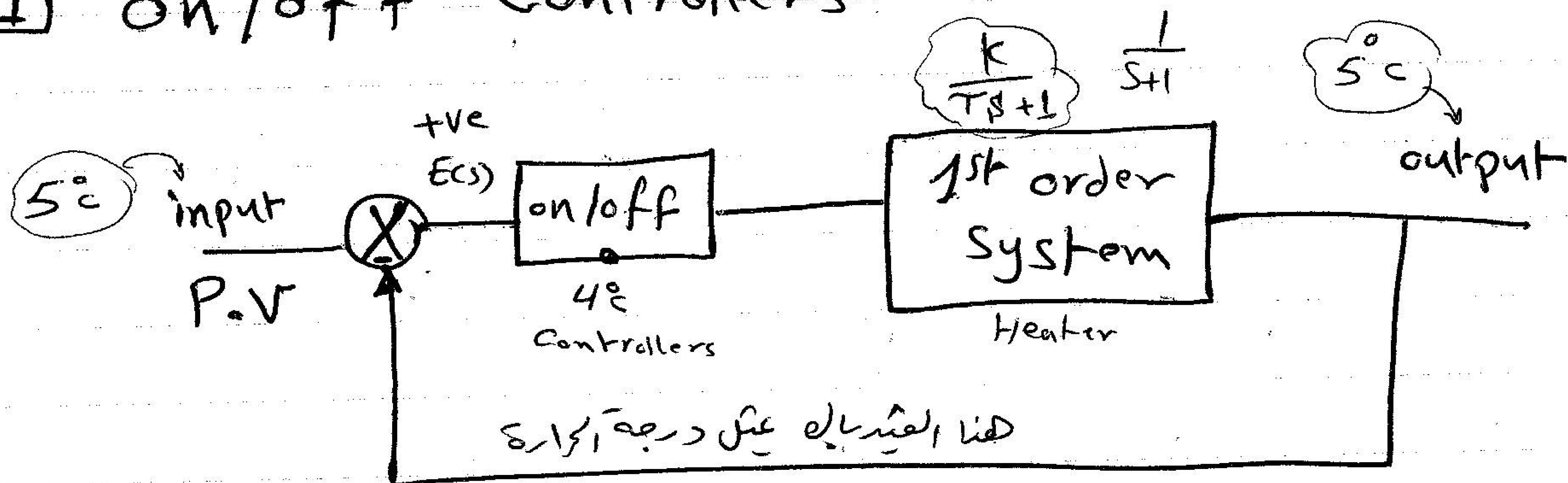
CONTROLLERS

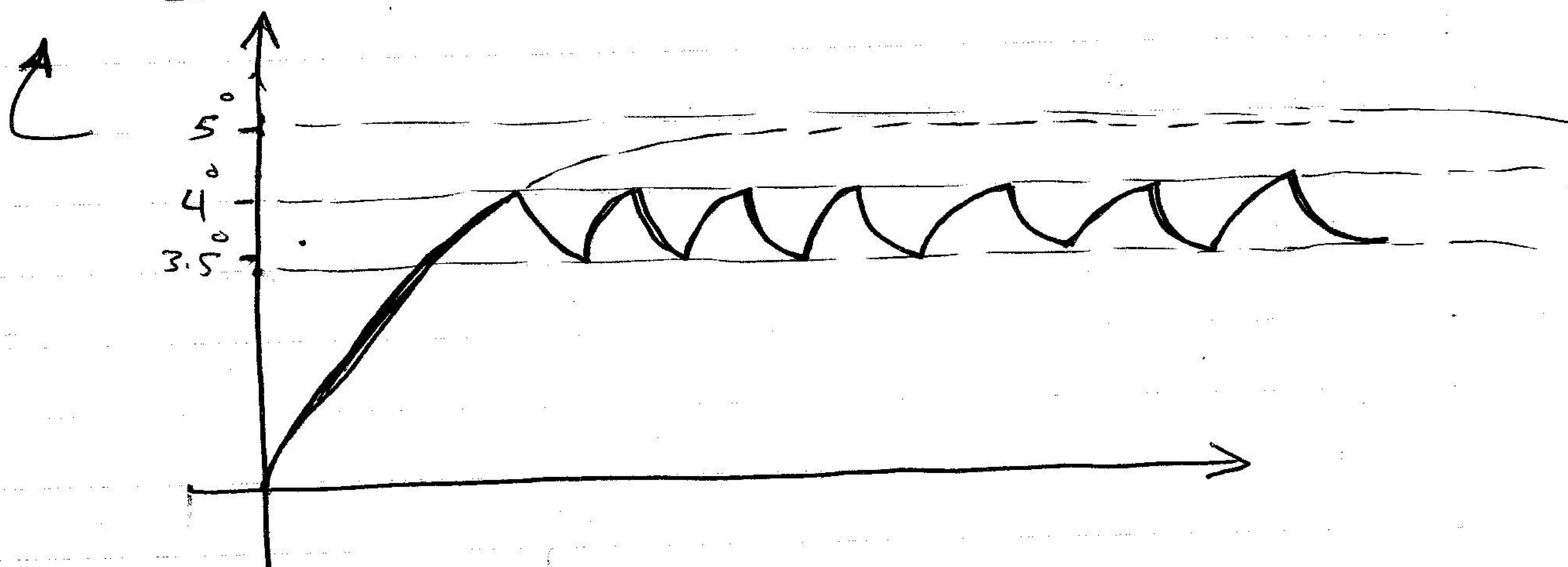




There is no Controller here!

1 on/off Controllers

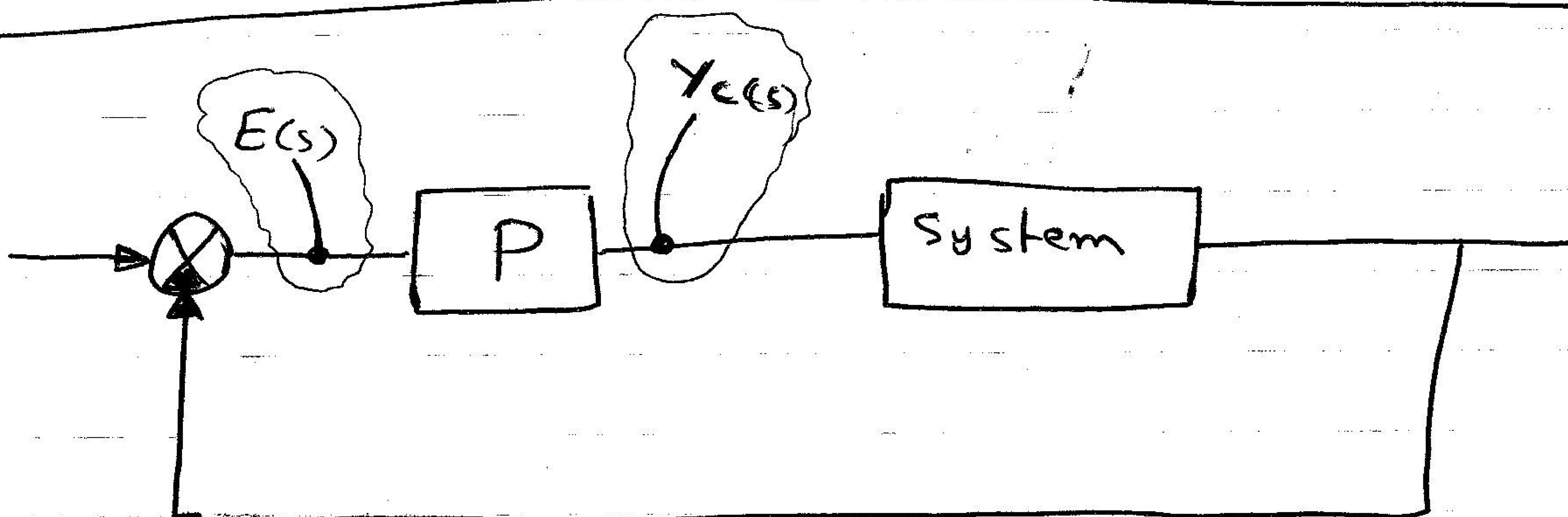




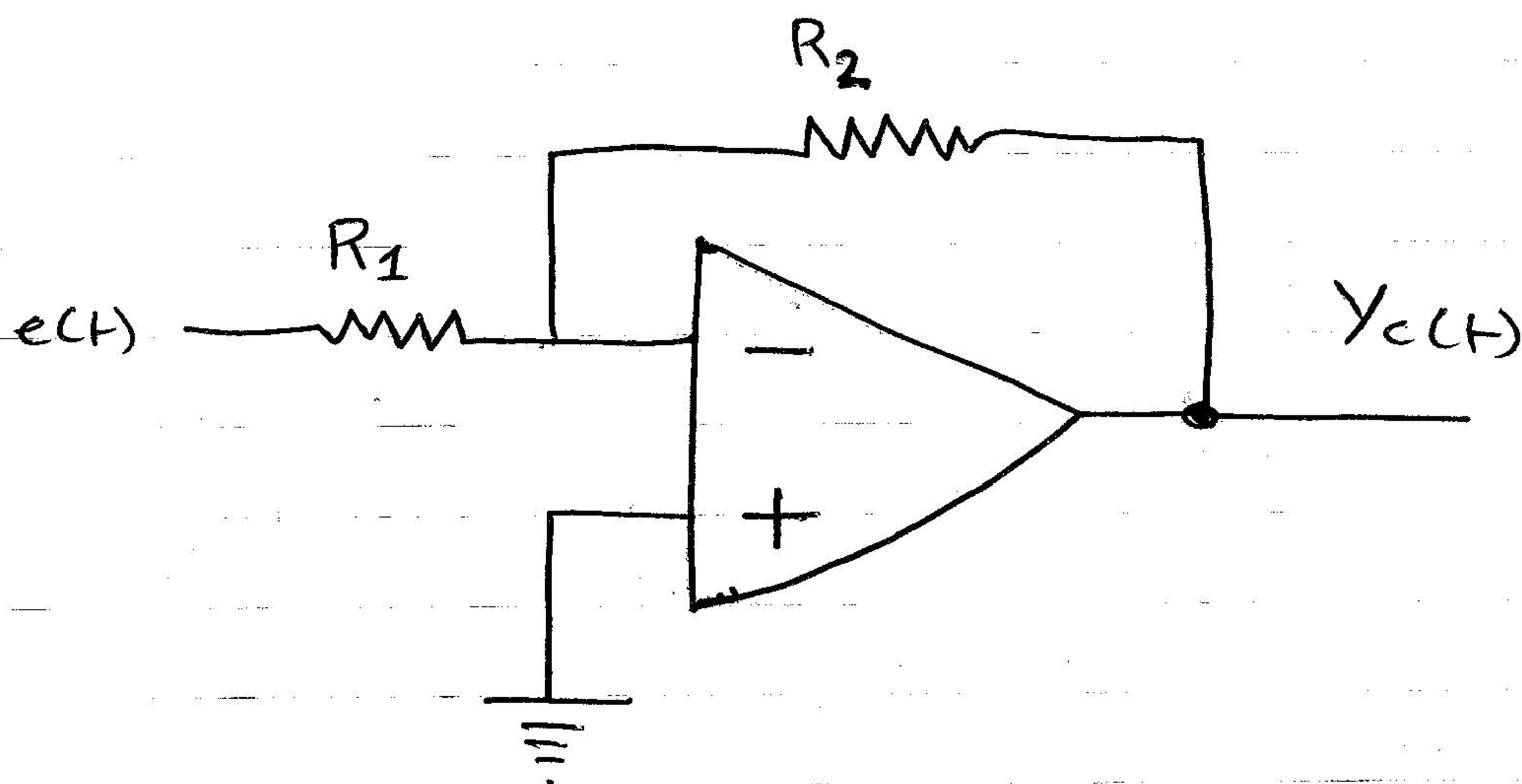
An Example on (on/off) Controllers : Thermo Switch

[2] Analogue Controllers

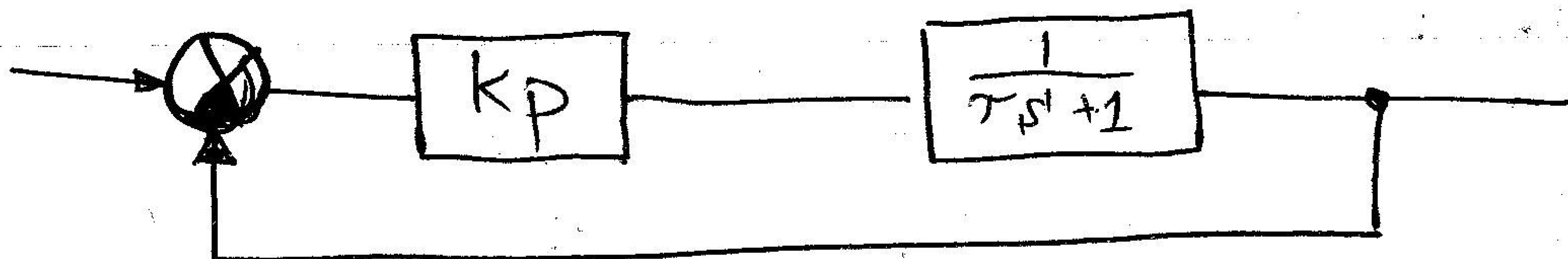
[a] Proportional Controller [P] : "الحاكم التناسبي"



$$Y_c(s) = K_p * E(s)$$



$$y_c(t) = \frac{-R_2}{R_1} * e(t) \implies K_p = \frac{R_2}{R_1}$$



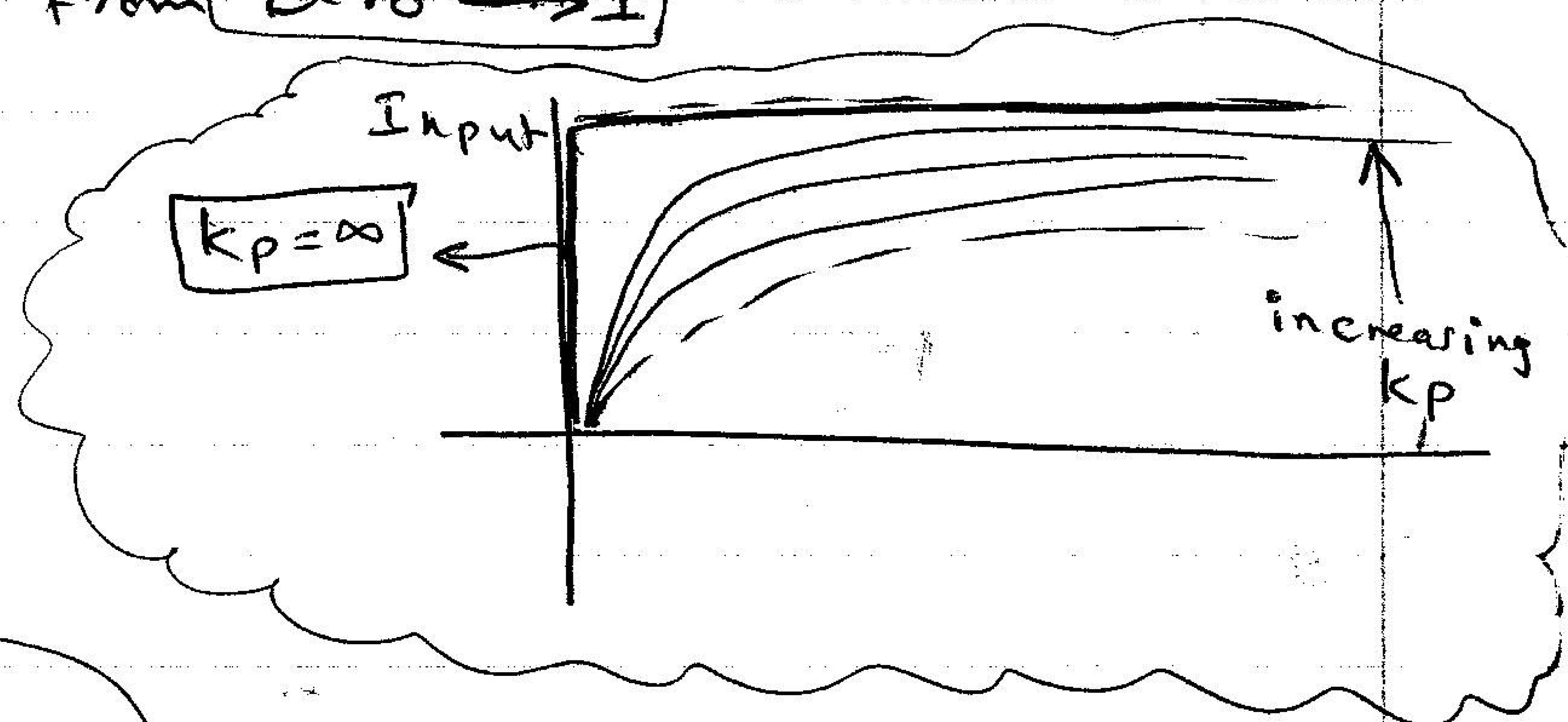
$$\equiv \frac{\frac{k_p}{\tau s+1}}{1+\frac{k_p}{\tau s+1}} = \boxed{\frac{k_p}{\tau s+1+k_p}} = \boxed{\frac{\frac{k_p}{1+k_p}}{\frac{\tau}{1+k_p}s+1}}$$

$$\circledast \quad \boxed{\text{Total gain} = \frac{k_p}{1+k_p}}$$

⊛ if kp increased from [Zero → ∞]

⊛ The total gain increases from [Zero → 1]

$$\boxed{Error = \frac{1}{1+k_p} R}$$



* kp = 10

⇒ $gain = \frac{10}{1+10} = \frac{10}{11} = 0.909$

* kp = 20

⇒ $gain = \frac{20}{1+20} = \frac{20}{21} = 0.95$

ميزات الحاكم التناسبي

ميزاته
1. رخيص الثمن
2. سريع الاستجابة
3. بسيط وسهل التركيب

سيئاته
1. لا يلغي إشارة الخطأ

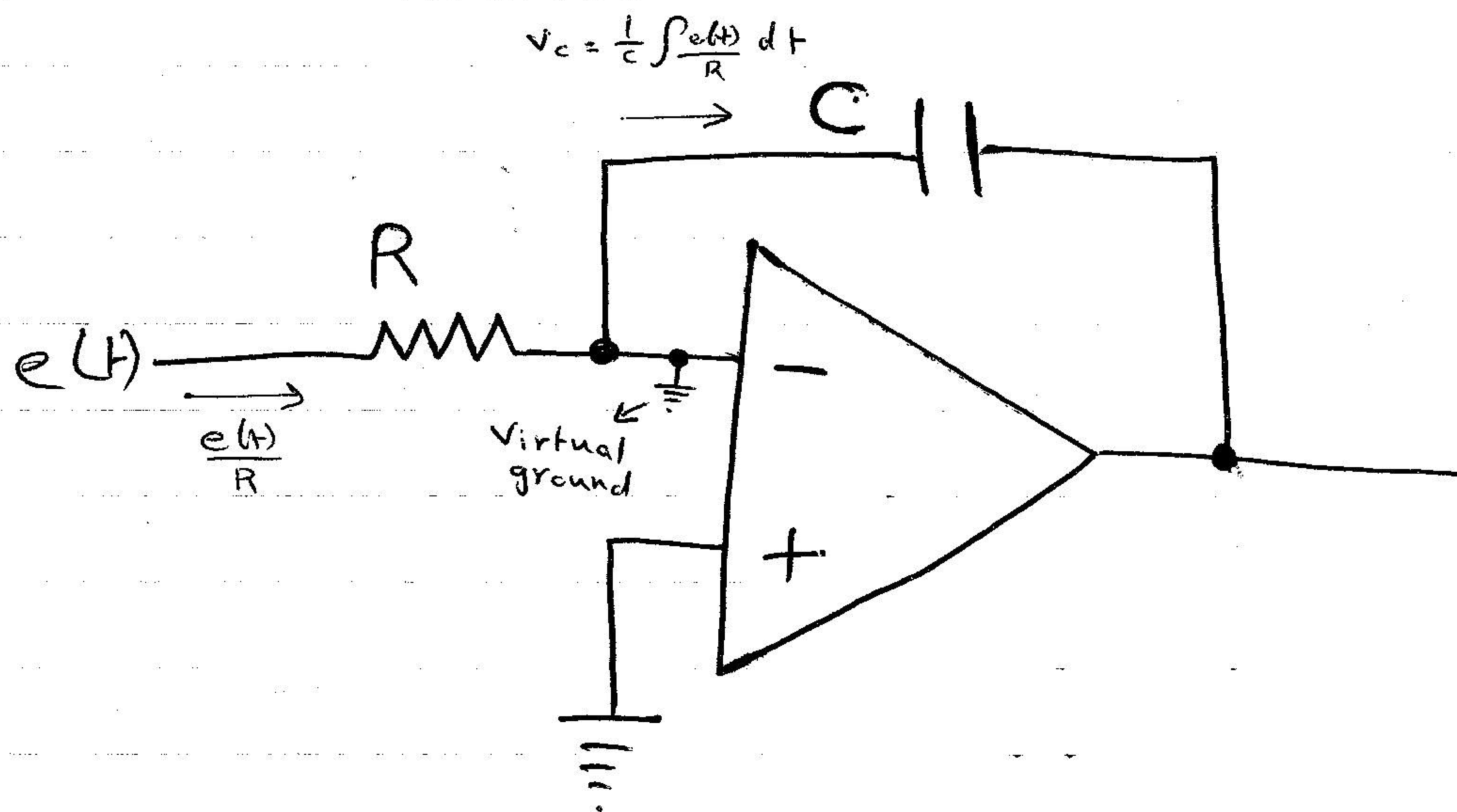
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[b] Integral Controller [I] "الحاكم التكاملي"

$$\boxed{y_c(t) = k_i \int e(t)\, dt}$$

$$Y_c(s) = k_i * \frac{1}{s} E(s) \Rightarrow \boxed{Y_c(s) = \frac{k_i}{s} * E(s)}$$

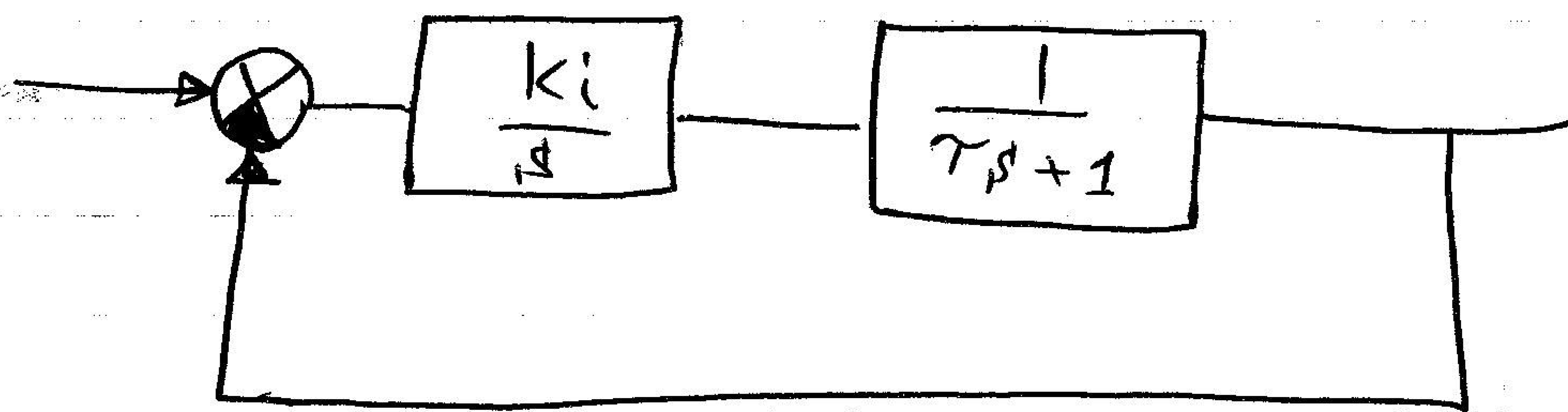
$$T.F = \frac{Y_c(s)}{E(s)} = \frac{k_i}{s}$$



$$V_c = \frac{1}{c}\int \frac{e(t)}{R}\, dt = \frac{1}{Rc}\int e(t)\, dt$$

$$\Rightarrow V_o = \frac{-1}{Rc}\int e(t)\, dt$$

$$\therefore k_i = \frac{1}{Rc}$$



$$\frac{\frac{k_i}{s(\tau s+1)}}{\frac{k_i}{s(\tau s+1)} + 1} = \frac{k_i}{k_i + s(\tau s + 1)} = \frac{k_i}{\tau s^2 + s + k_i}$$

$$= \frac{\frac{k_i}{\tau}}{s^2 + \frac{s}{\tau} + \frac{k_i}{\tau}}$$

$$\frac{k\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$ → 2nd order Equation

⇒ total gain = 1

Error = 0

ميزات الحاكم التكاملي

ميزاته:
1. يُلغي إشارة الخطأ نهائياً
2. بسيط التركيب
3. رخيص الثمن

سيئاته:
1. يمكن أن يُحدث اهتزازات في النظام أو يزيد من قيمة الاهتزاز.
2. يمكن أن يؤثر على استقرار النظام.

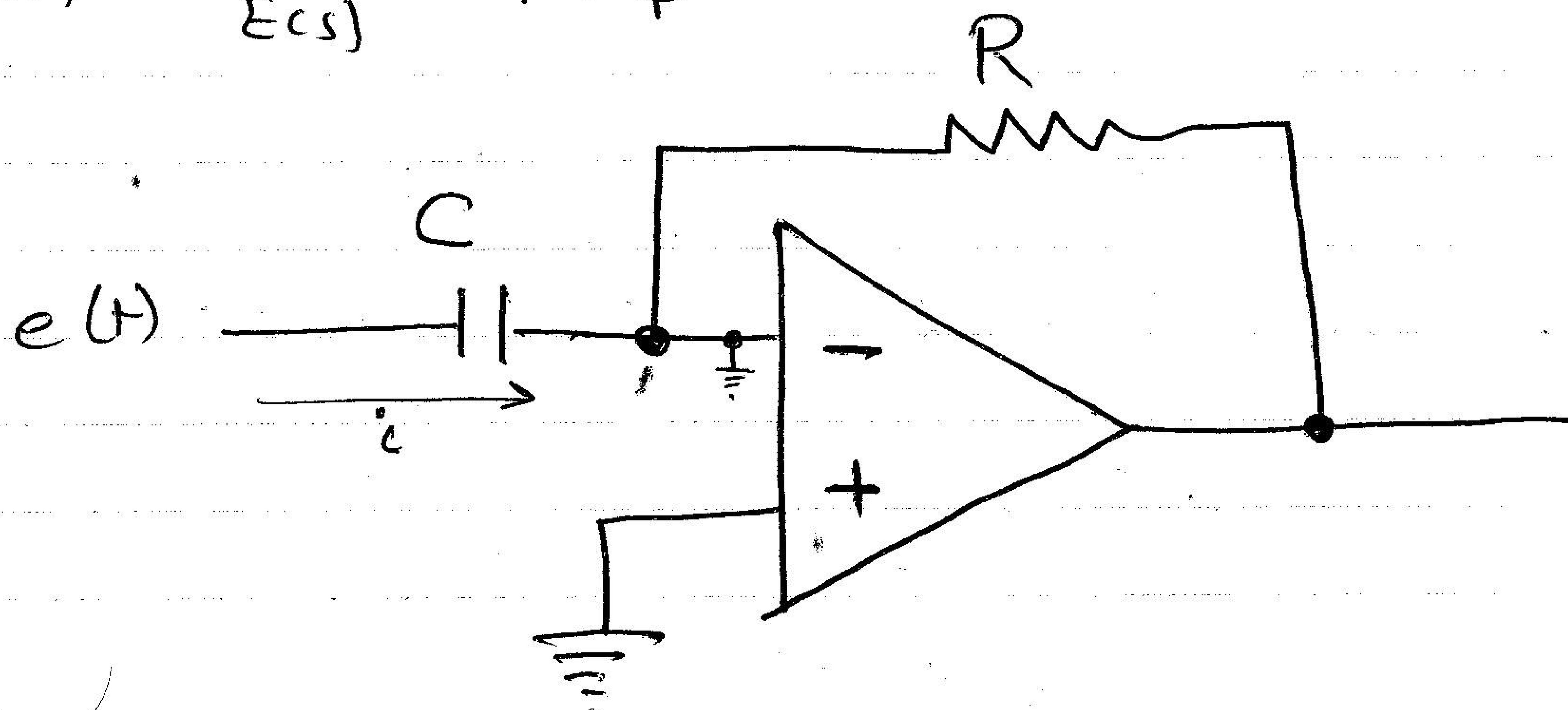
الخميس 31/10

C Derivative Controller [D] "الحاكم التفاضلي"

$$y_c = K_d * \frac{de(t)}{dt}$$

$$Y_c(s) = K_d\, s\, E(s)$$

$$G_c(s) = \frac{Y_c(s)}{E(s)} = K_d s$$



⊛ $V_c = \frac{1}{C}\int i\,dt$

$e(t) = \frac{1}{C}\int i\,dt$

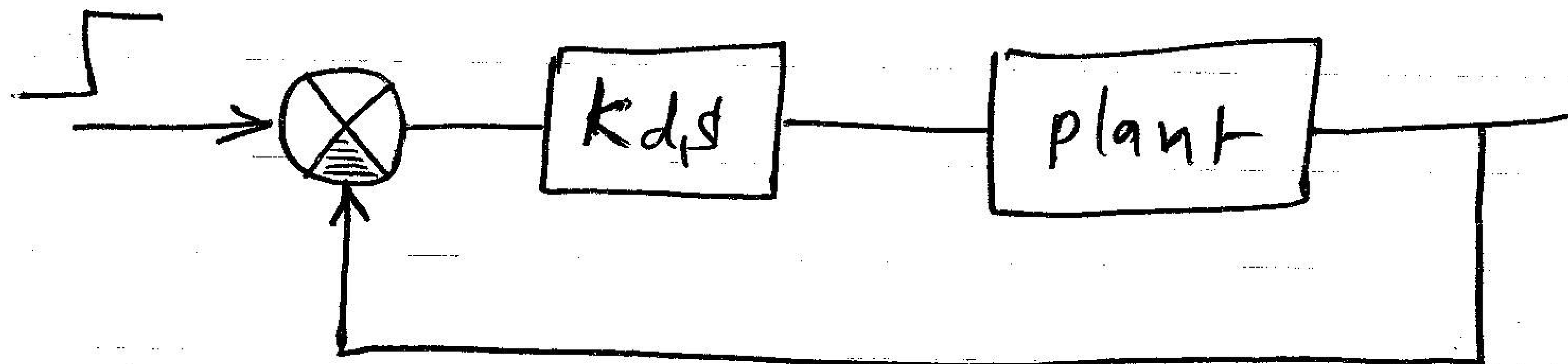
$C\,e(t) = \int i\,dt$

$C\frac{de(t)}{dt} = i$ ---- ⊛

⊛ $V_R = Ri = RC\frac{de(t)}{dt}$

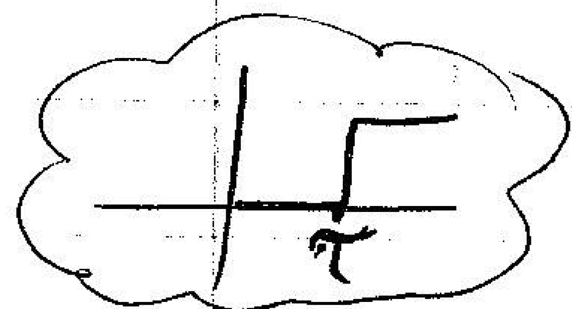
⊛ $V_o = -RC\frac{de(t)}{dt}$

$\Rightarrow$ $\boxed{K_d = RC}$



derivative of this signal [step] is [impulse] pulse

take care from this Controller!! :-)



⊛ D-Controller when added to P-Controller, provides ameans of high sensitivity.

"ميزات هذا الكائن"

⊛ an advantage of using D-Controller is that it responds to the change of actuating error & can provide a significant Correction before the magnitude of the error goes Large.

and thus, it tends to increase the stability of the system.

why we do not use D-Controller alone?

① it can't eliminate error.

② in the case of unit step input, this controller give an impulse function with amplitude goes to (∞)!

d PID - Controller

Combines the advantages of P+I+D Controllers.

$$y_c(t) = K_p * e(t) + K_i \int e(t)\,dt + K_D \frac{de(t)}{dt}$$

$$Y_c(s) = K_p E(s) + \frac{K_i}{s} E(s) + K_D s\, E(s)$$

$$= E(s)\left[K_p + \frac{K_i}{s} + K_D s\right]$$

$$\therefore G(s) = \frac{Y_c(s)}{E(s)} = K_p + \frac{K_i}{s} + K_D s$$

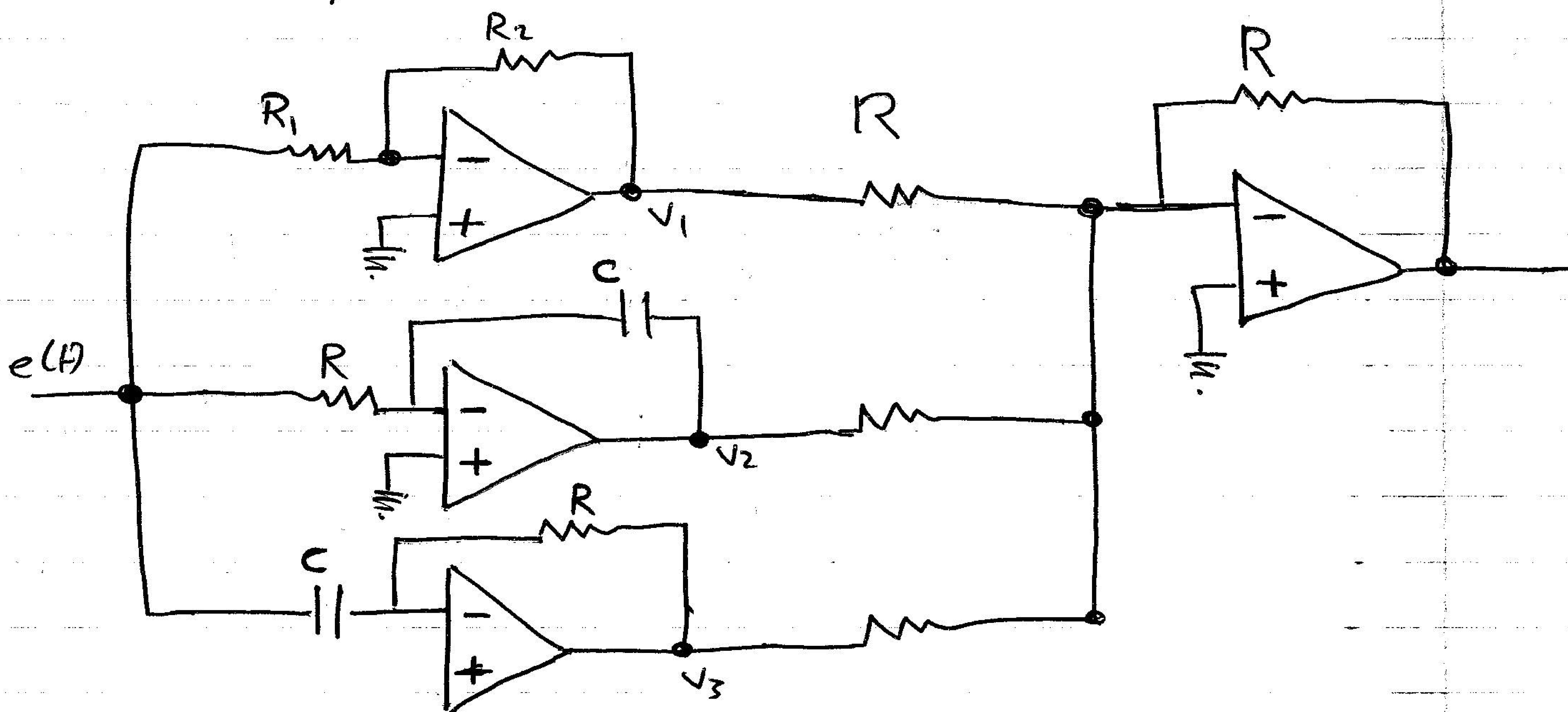
$$\therefore G(s) \; (\text{as T.F}) = \frac{K_D s^2 + K_p s + K_i}{s}$$

$$G(s) = K_p\left[1 + \frac{1}{T_i s} + T_d s\right]$$

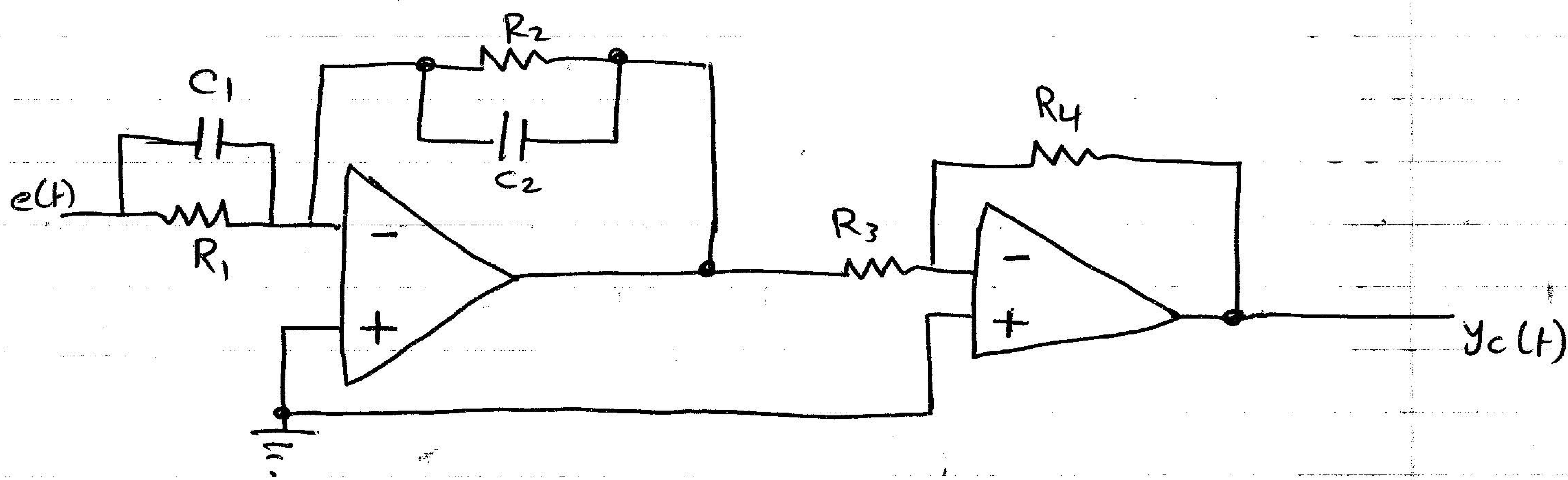
→ Some engineers use this equation.

من مساوئ هذا النظام: صعوبة عملية ضبطه، لسبب احتوائه على ثلاثة متغيرات P، I، D.

⊛ PID Equivalent electronic Circuit :-



⊛ Another Equivalent ckT for PID-Controller (Ogata) :-

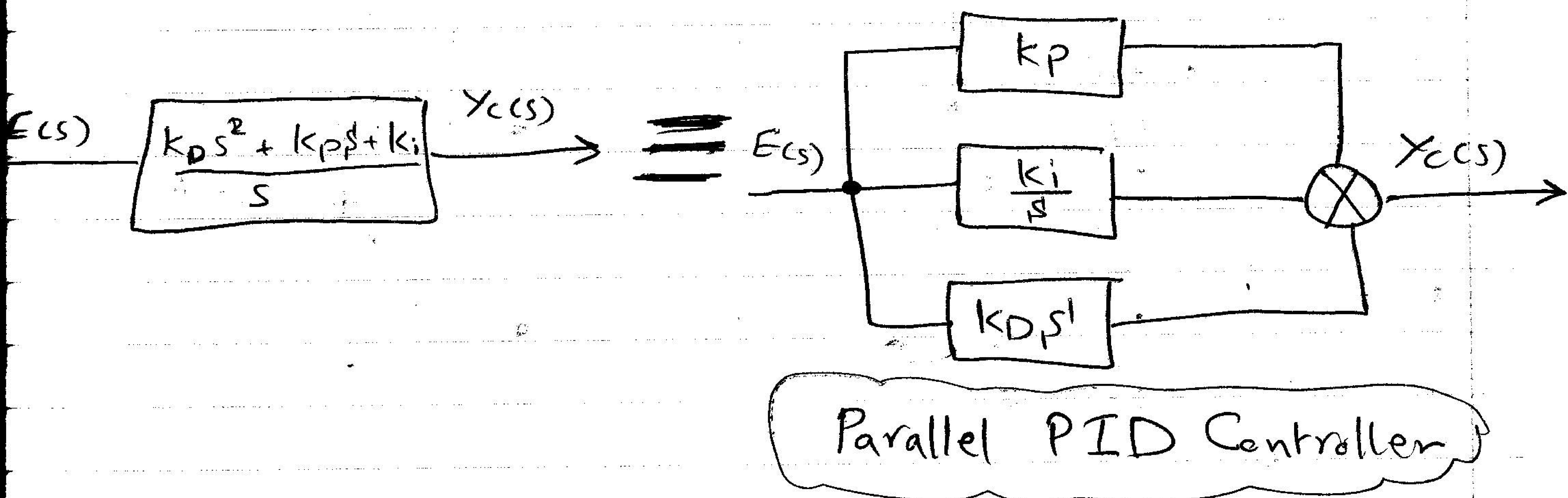


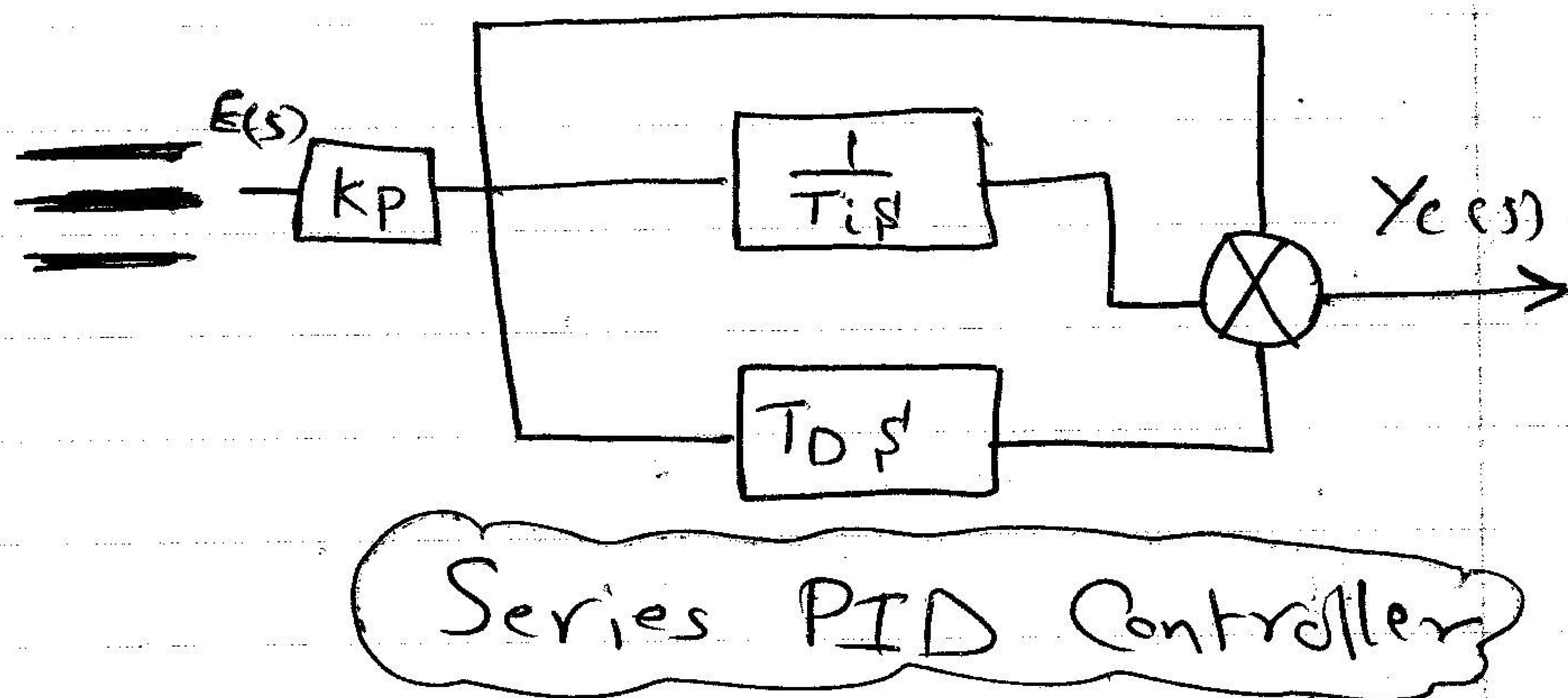
$$\frac{Y_c(s)}{E(s)} = G(s) = \frac{R_4}{R_3} * \frac{R_2}{R_1} \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_2 C_2 s}$$

هنا التحكم غير مستقل، التحكم بأي نوع يؤثر على الأنواع الأخرى

11/3 الأحد

PID Controller

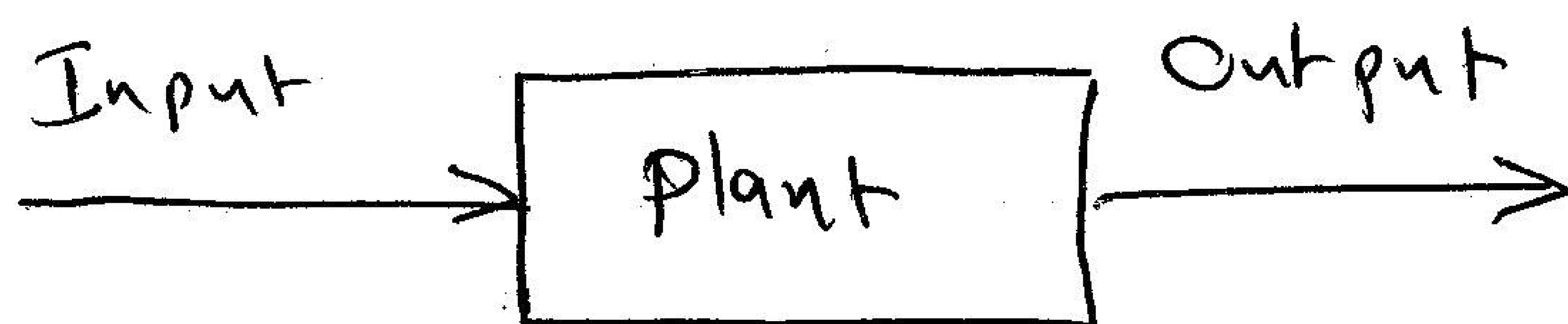




Tuning of PID Controller [Using Ziegler-Nicolas] Methods

[1] The 1st Method (Reaction Line):

in This method, we obtain experimentally the response of the plant to a unit step input (2nd order system).

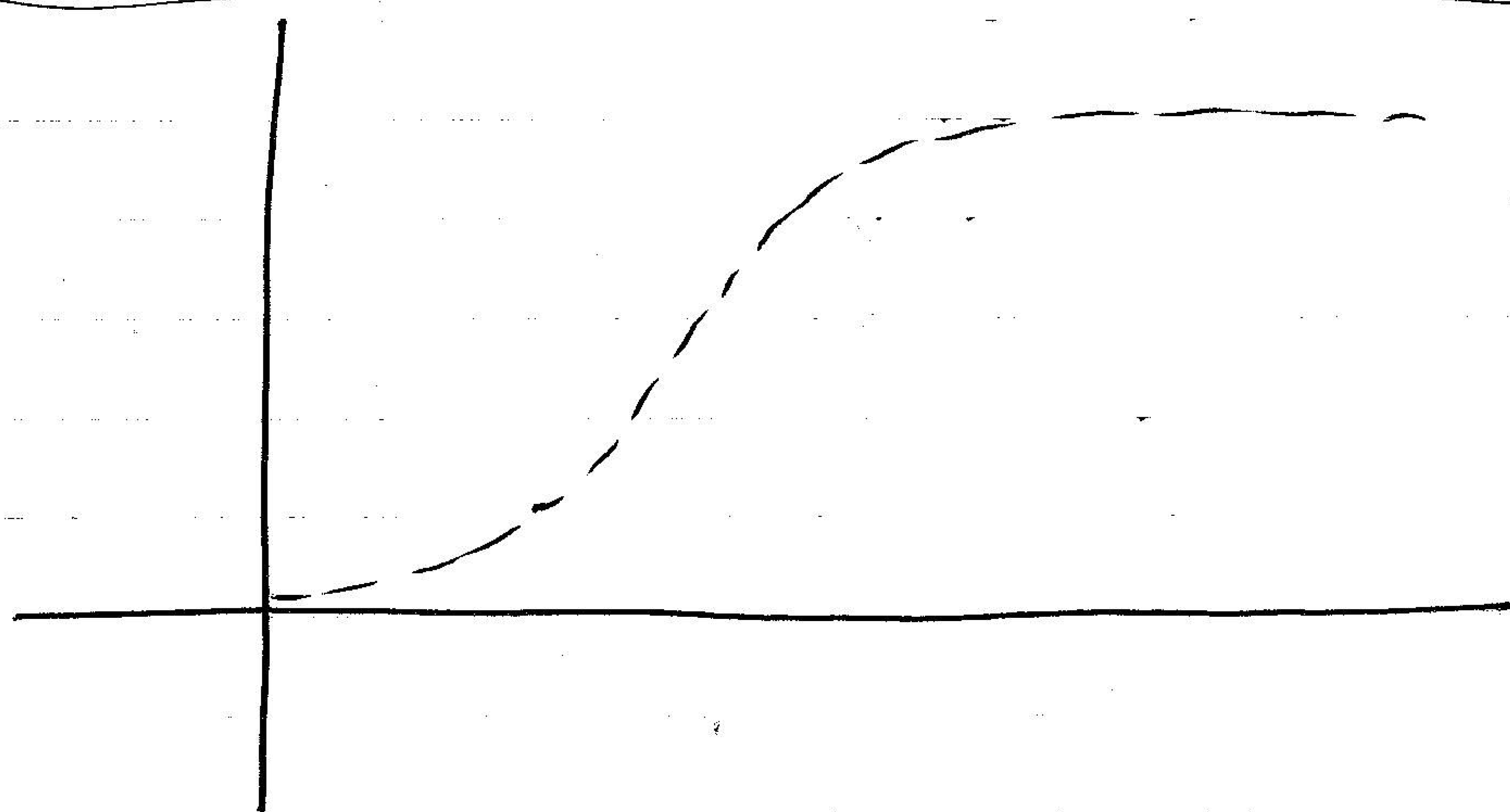


No feed back
No Controller

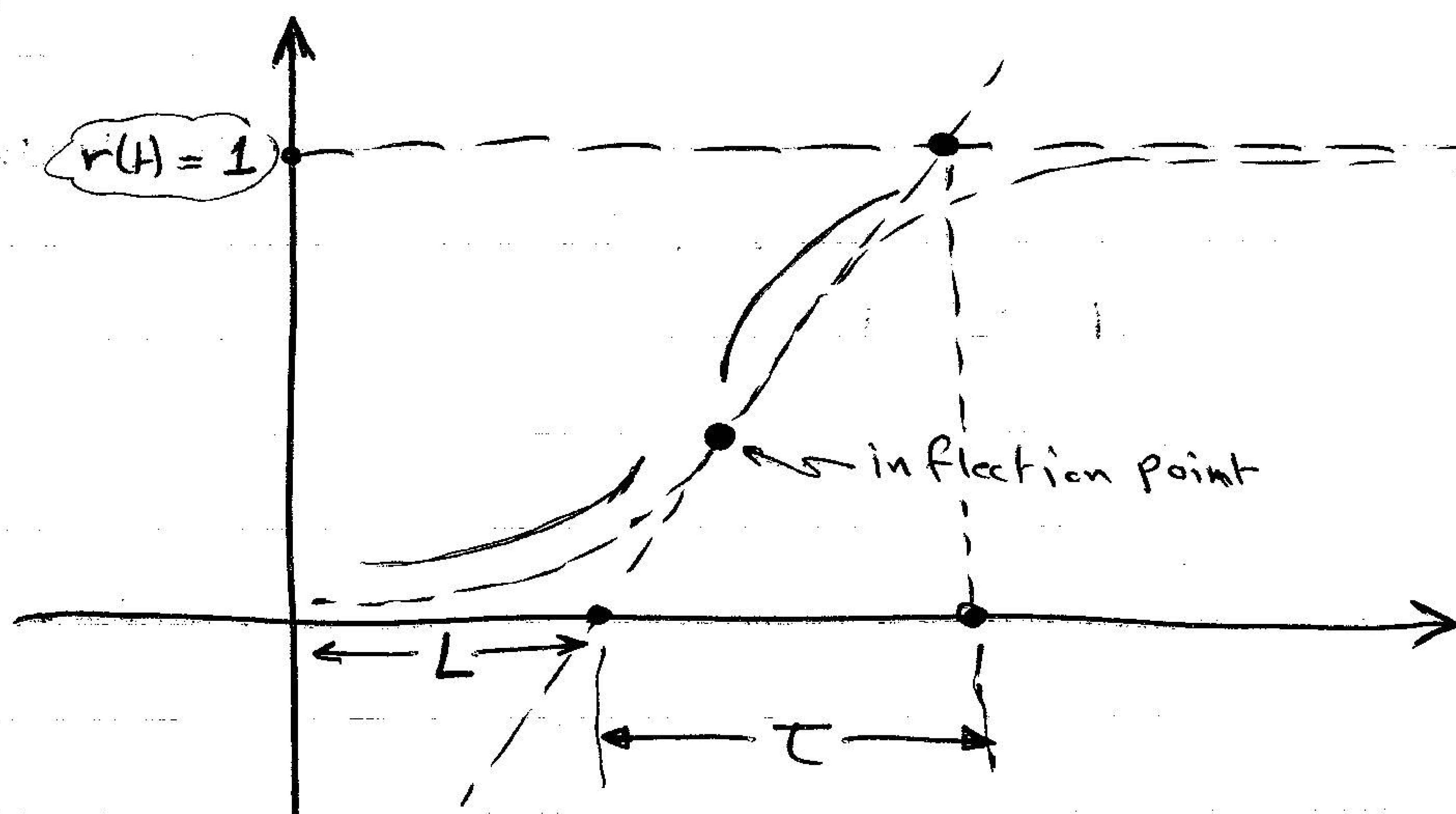
* If the plant involves neither integrators nor dominant complex conjugate poles, then the step response curve may looks as S-shaped.

* $\frac{1}{S(S+5)}$ ✗ → Wrong, because there is an integrator $\left(\frac{1}{S}\right)$.

$\frac{1}{S^2+S+1}$ ✗ → Wrong, because there are complex conjugate poles.



* The delay time & time constant are determined by drawing a tangent line at the inflection point.



L : delay time

τ : time constant

Type of Controller	K_p	τ_i	τ_d
P	$\frac{T}{L}$	∞	Zero
PI	$0.9\,\frac{T}{L}$	$\frac{L}{0.3}$	Zero
PID	$1.2\,\frac{T}{L}$	$2L$	$0.5L$

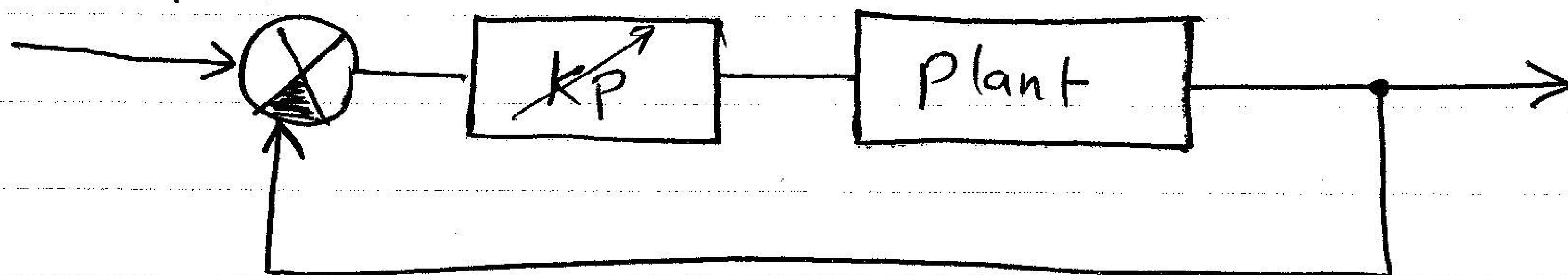
apply to Series PID Controller

* $K_i = K_p * \frac{1}{\tau_i}$
* $K_D = K_p * T_D$

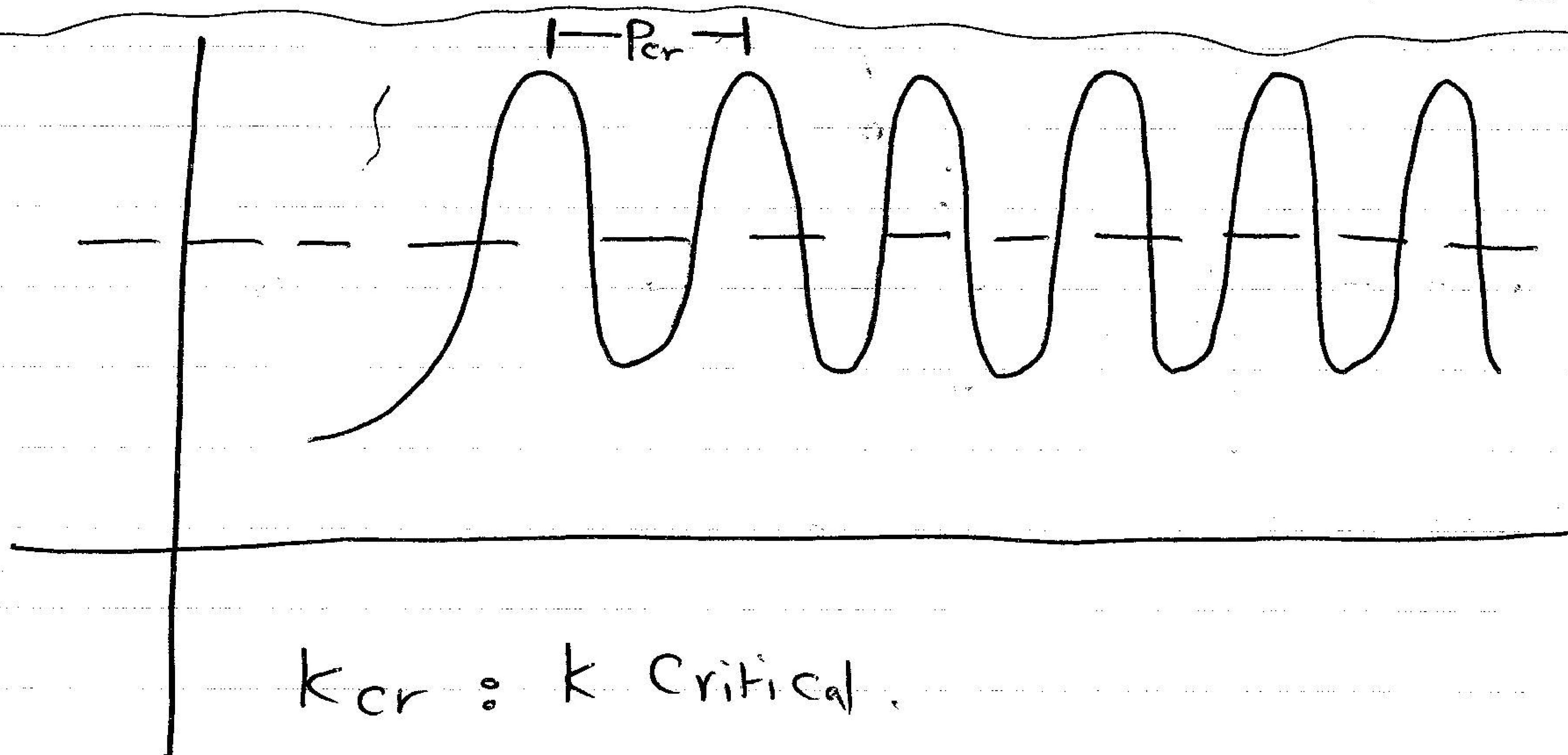
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[2] The 2nd Method (Oscillation Rule):

Using the proportional Controller only [P-Controller]



* Increase k from Zero to Critical Value **Kcr** at which the output first exhibits sustained Oscillation.



K_{cr} : K Critical.

P_{cr} : Peak Critical.

Type of Controller	Kp	Ti	Td
P	0.5 Kcr	∞	Zero
PI	0.45 Kcr	$\frac{1}{1.2}$ Pcr	Zero
PID	0.6 Kcr	0.5 Pcr	0.125 Pcr

11/5 الأحد

Stability of Linear feedback System [Dorf ch6]

- a stable system should exhibit a bounded output if the corresponding input is bounded.

- So, a stable System is a dynamic system with a bounded system to a bounded input.

- The stability of a feedback system is directly related to location of the roots of the characteristic equation of the total transfer function $G_T(s) = \frac{\text{output}}{\text{Input}}$

Stability Checking Methods

[1] Direct Method

In this method, the roots of ch. eq. should be calculated & assigned to the S-plane.

System is said to be "Stable" if all roots located at the left side of S-plane, and one +ve root is enough to make the whole system "Unstable"

$G_T(s) \equiv T(s)$ → Total T.F. نفس بعض

e.g : check the stability of the following systems :-

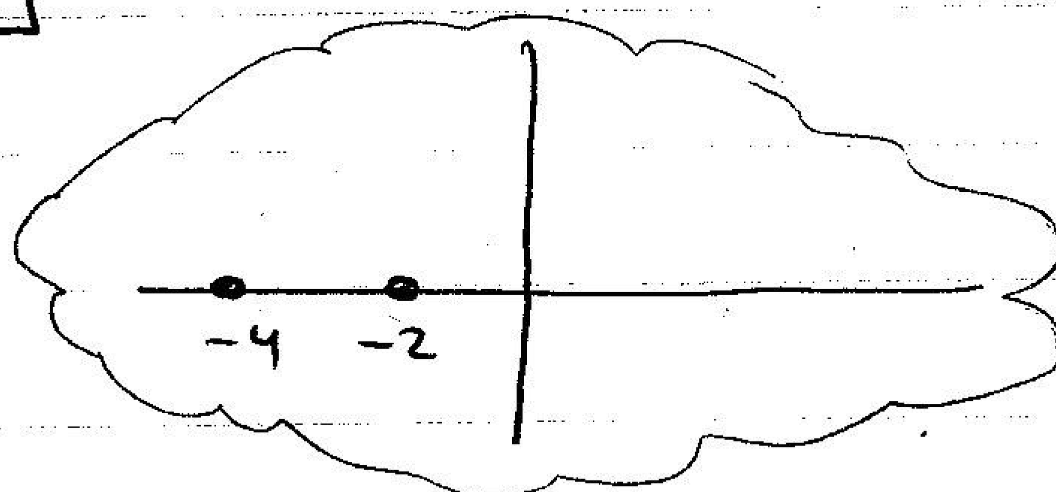
a $G_T(s) = \dfrac{22s}{s^2+6s+8}$

$\Rightarrow q(s) = s^2+6s+8 = 0$

$(s+4)(s+2) = 0$

$\boxed{s=-4}$ $\boxed{s=-2}$

∴ The system is stable



---

b $y(s) = \dfrac{1}{s^2-1}$; $x(t) = e^{t}$

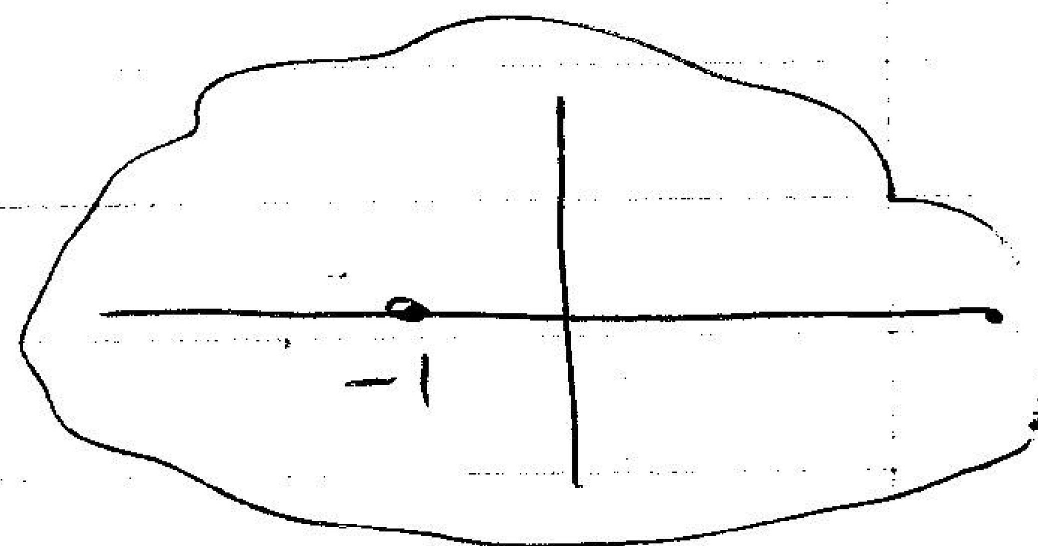
↓

$X(s) = \dfrac{1}{s-1}$

$G_T(s) = \dfrac{Y(s)}{X(s)} = \dfrac{\frac{1}{s^2-1}}{\frac{1}{s-1}} = \dfrac{\frac{1}{(s-1)(s+1)}}{\frac{1}{(s-1)}} = \dfrac{1}{(s-1)(s+1)} * (s-1)$

$= \dfrac{1}{s+1}$

∴ $q(s) = s+1 = 0 \Rightarrow \boxed{s=-1}$



∴ The system is stable.

---

c $T(s) = \dfrac{3}{s^3+1}$

$q(s) = s^3+1 = 0$

$q(s) = s^3+1 = 0$

$q(s) = (s+1)(s^2-s+1) = 0$

$$\begin{array}{r|l} & s^2 - s + 1 \\ s+1 & s^3 + 1 \\ & -s^3 \mp s^2 \\ \hline & -s^2 + 1 \\ & +s^2 + s \\ \hline & s+1 \\ & s+1 \\ \hline & 0 \end{array}$$

general formula

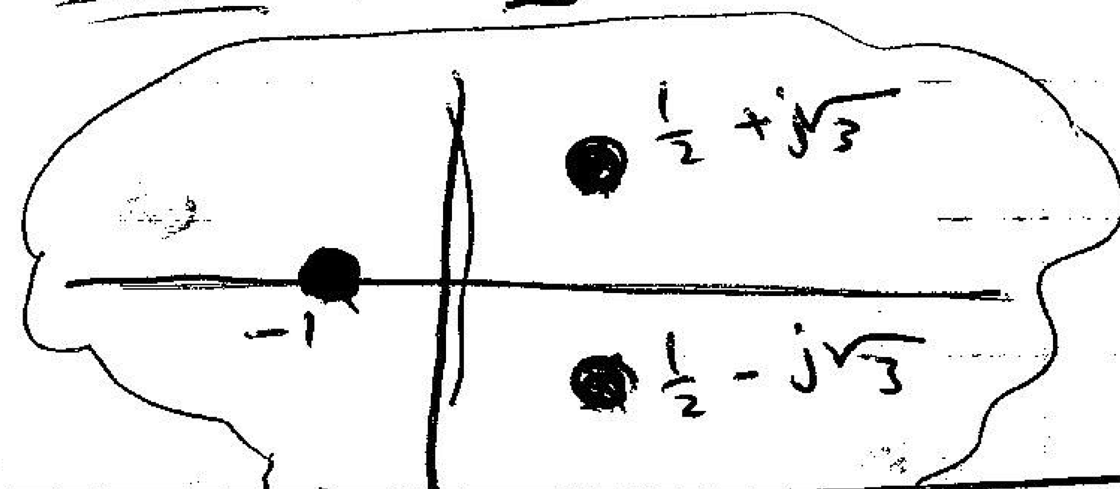
$s^2 - s + 1 = 0$

$$\boxed{\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$\frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm j\sqrt{3}}{2} \Rightarrow \left[\frac{1}{2} + j\sqrt{3} \,,\, \frac{1}{2} - j\sqrt{3} \right]$$

∴ Total roots : $\left[-1 \,,\, \frac{1}{2} + j\sqrt{3} \,,\, \frac{1}{2} - j\sqrt{3} \right]$

∴ The system is Unstable



2 Response to Impulse function

a linear system is stable if and only if the absolute value of its impulse response g(t) integrated over an infinite range is finite.

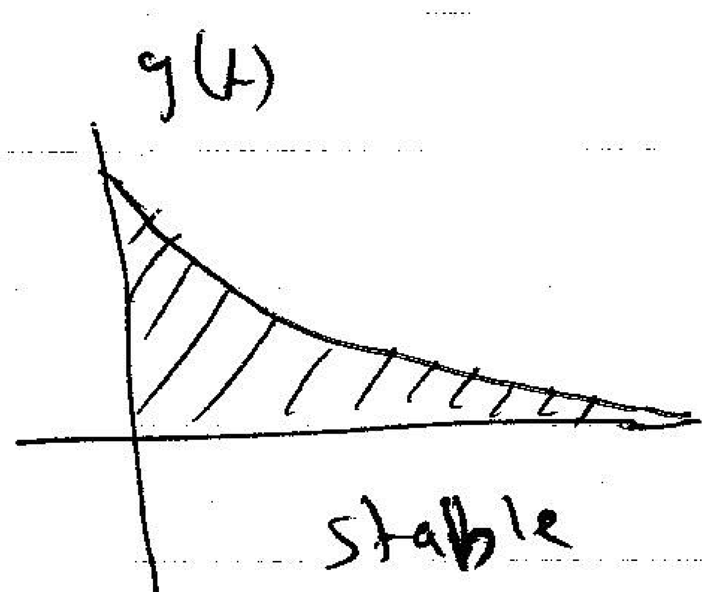


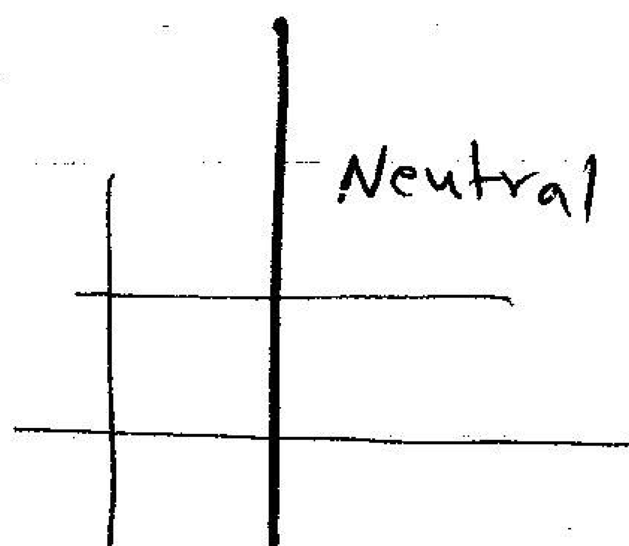
input	Lablace
δ(t)	1
1	1/s
t	$1/s^2$
t^2	$2/s^3$
t^3	$6/s^4$

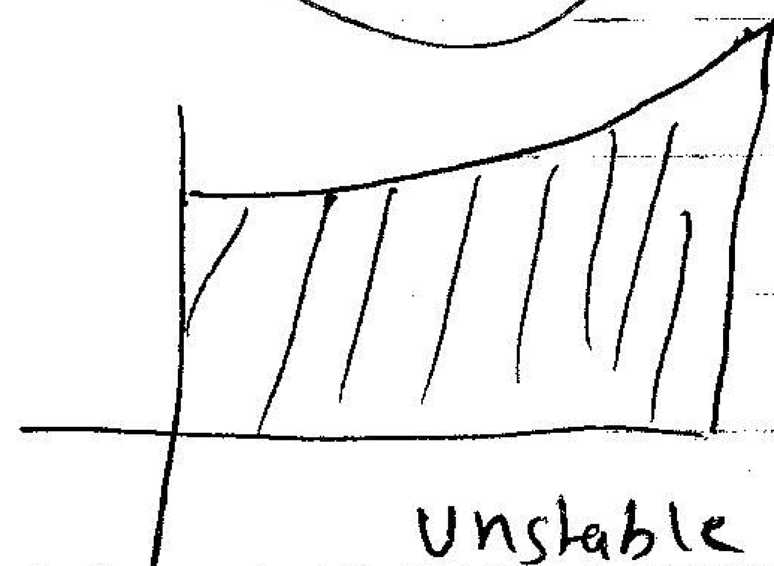
$$\int_0^\infty g(t)\,dt = \text{finite}$$

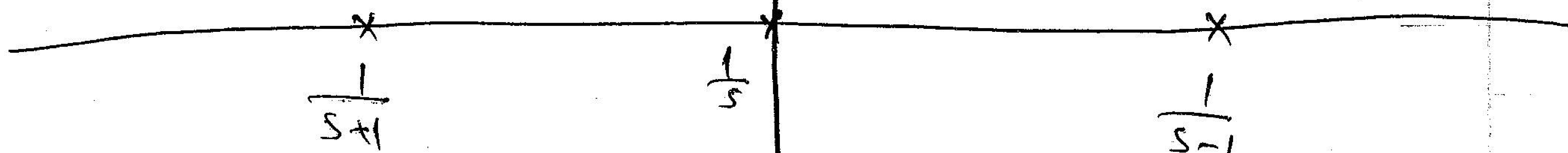
محدود (رقم)

1st order :

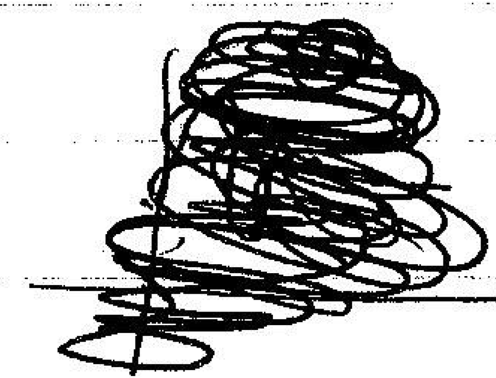


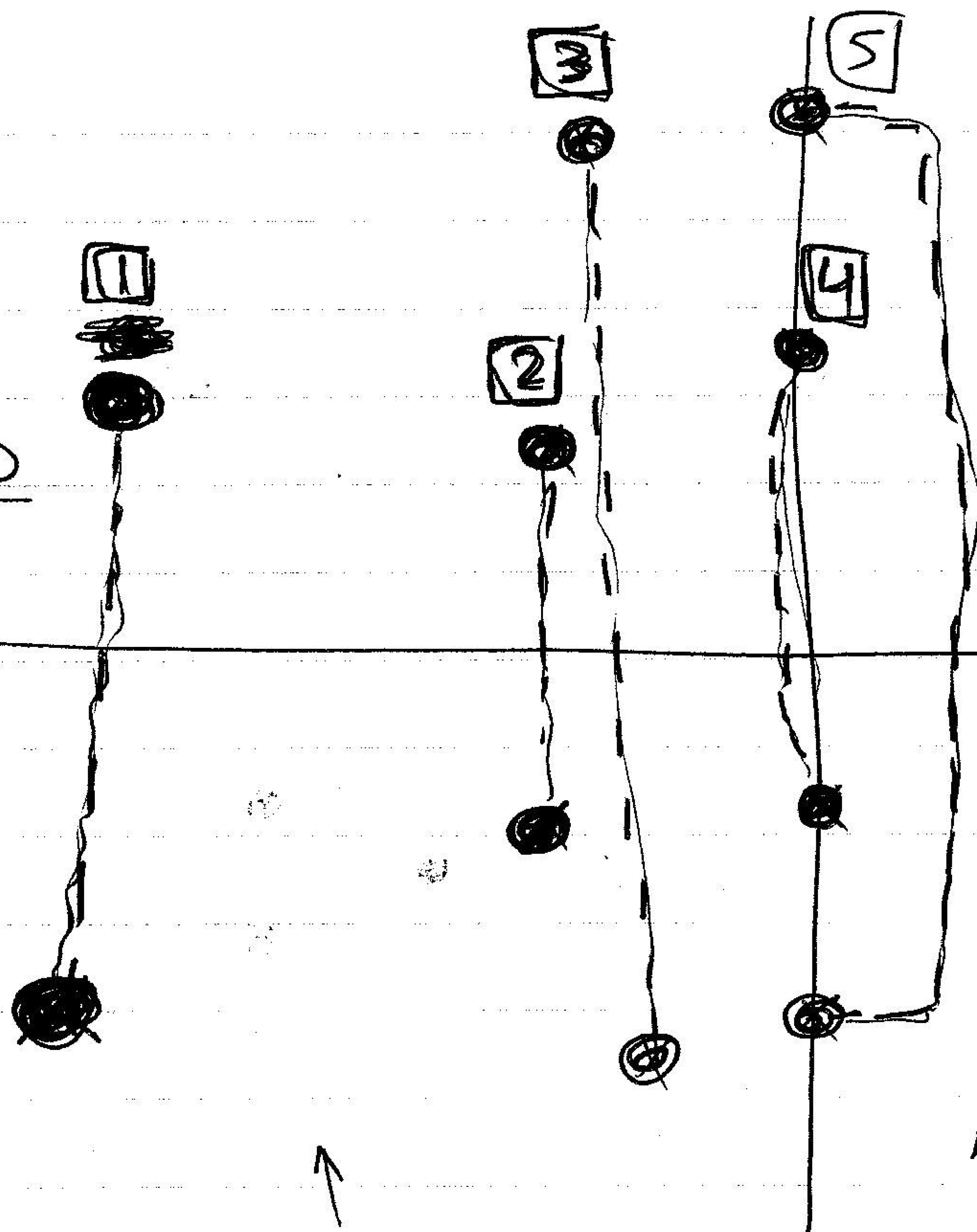


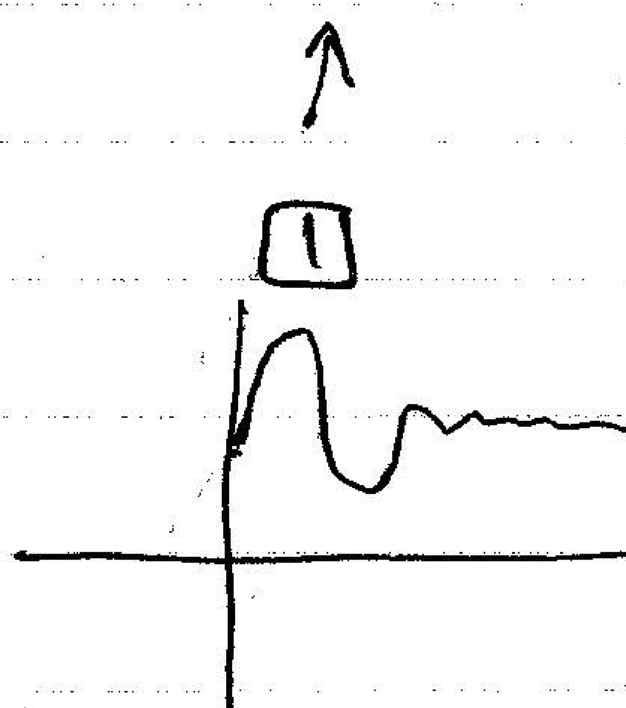


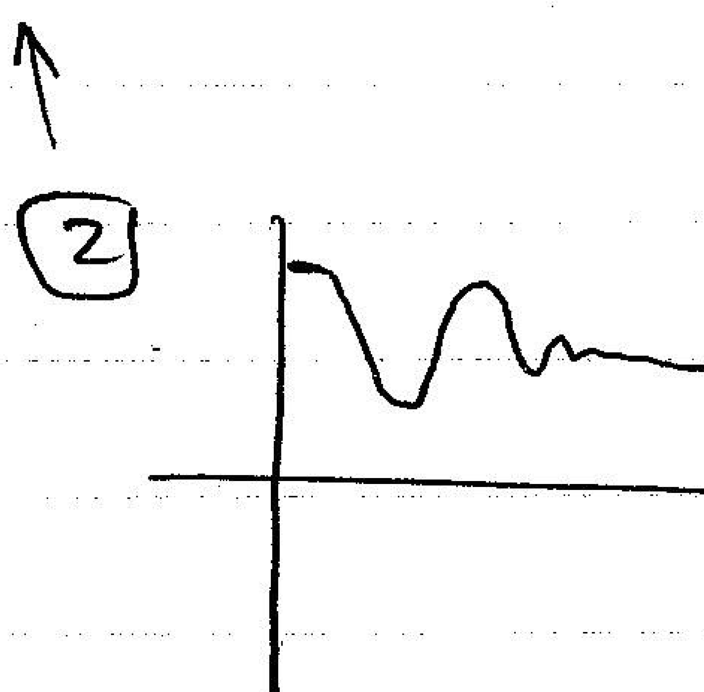


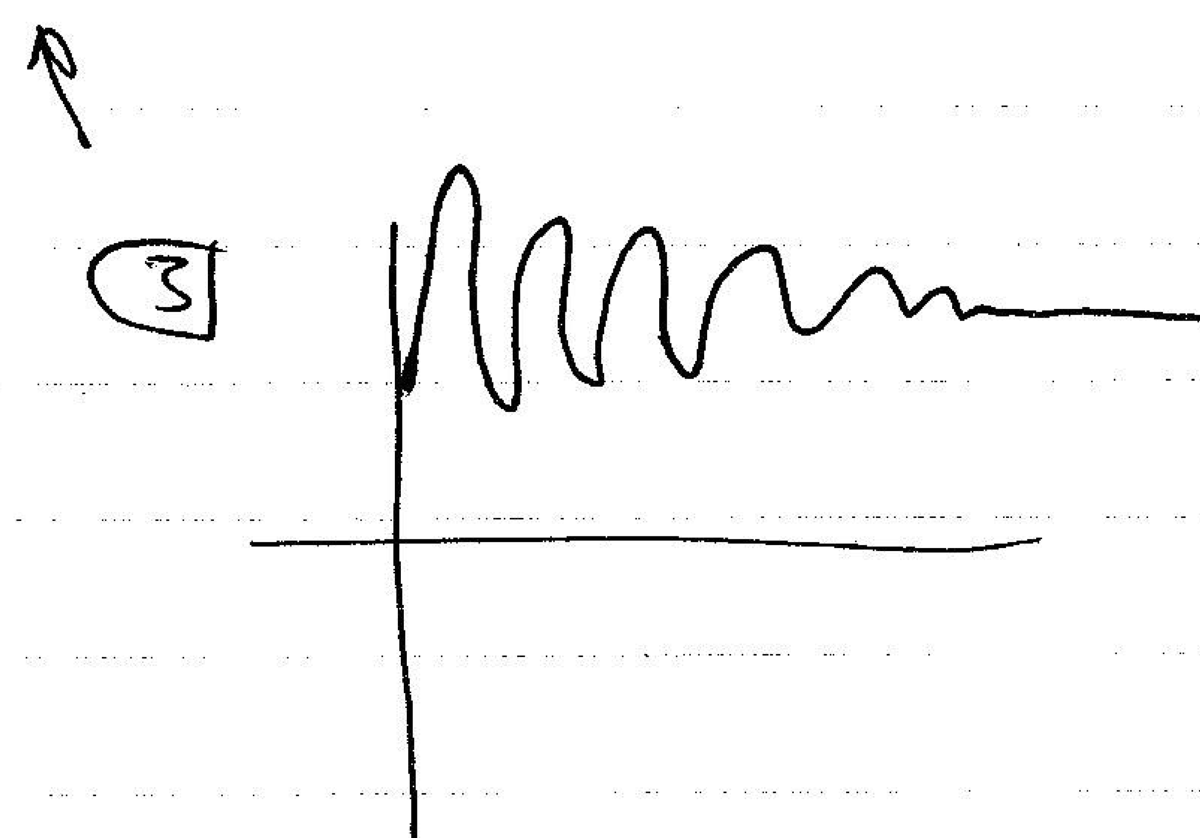
2nd order :

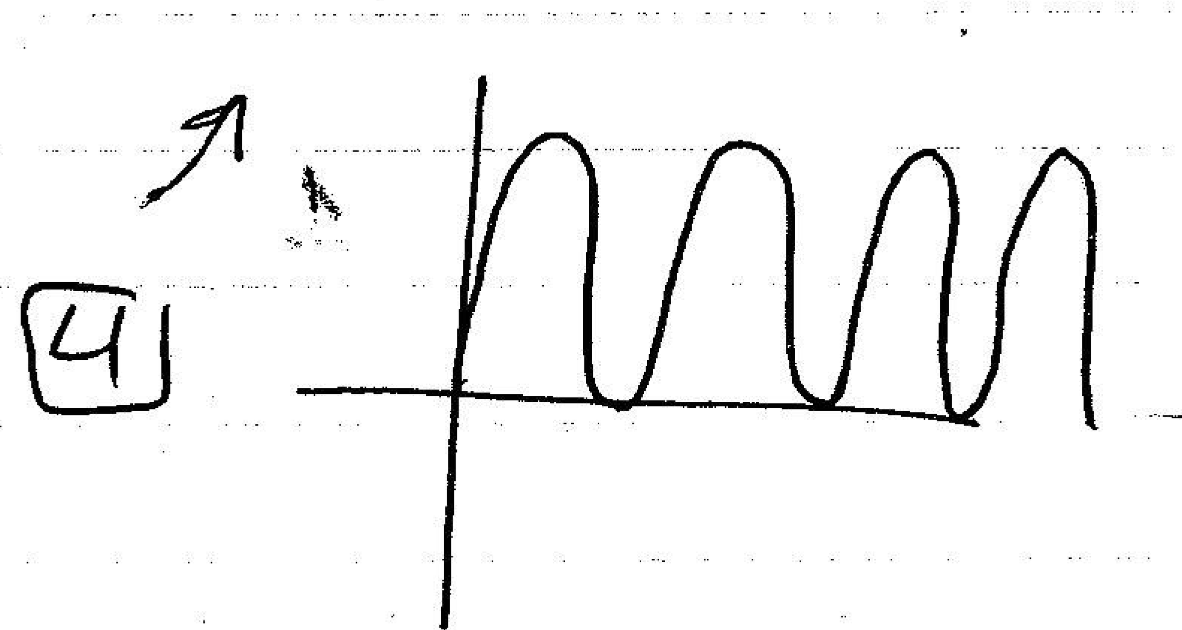


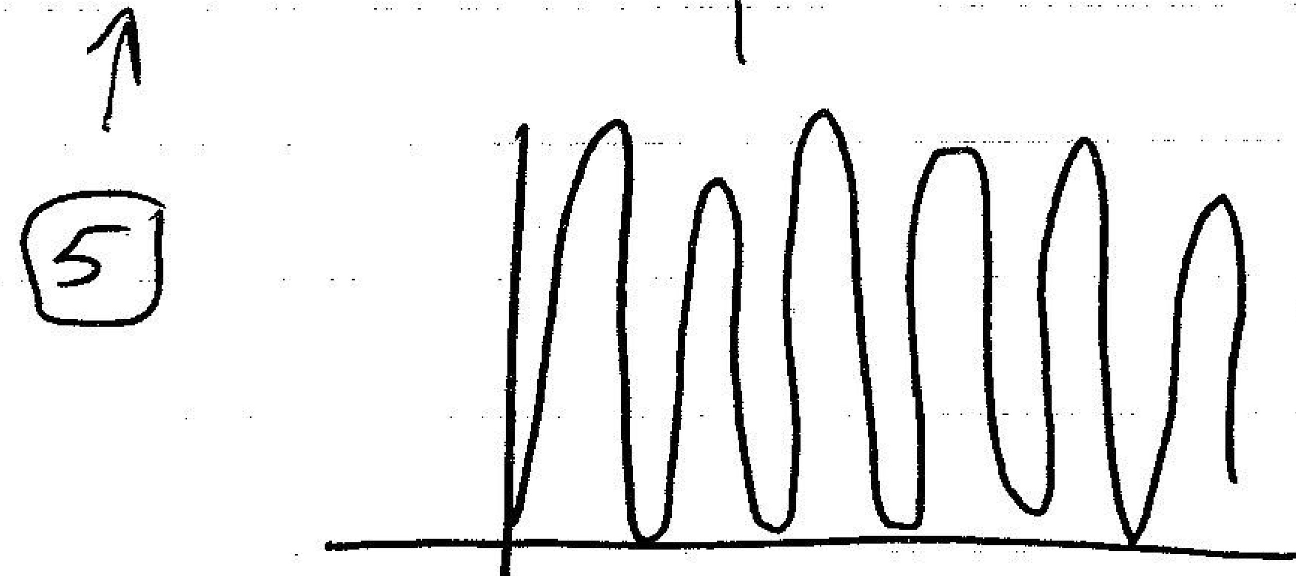












في 5 : يزيد التردد فقط

هذا صفر عند 4

3 Rouths- Hurwitz Stability Criteria :

- This technique allowed us to compute the number of roots of the characteristic equation in the right half plane without actually computing the values of roots.

- This gives us a design method for determining values of certain system that will lead to close loop stability.

- This method is based on ordering coefficients of the ch. eq. into an array as follows :-

$$q(s) = a_n S^n + a_{n-1} S^{n-1} + ----+ a_1 S^1 + a_0$$

S^n	a_n	a_{n-2}	a_{n-4}	---
S^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	---
S^{n-2}	b_{n-1}	b_{n-3}	b_{n-5}	---
|	C_{n-1}	C_{n-3}	C_{n-5}	---
|	⋮			
S	⋮			
1	h_{n-1}			

- $$b_{n-1} = \frac{a_{n-1} a_{n-2} - a_n a_{n-3}}{a_{n-1}} = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}$$

- $$b_{n-3} = \frac{a_{n-1} a_{n-4} - a_n a_{n-5}}{a_{n-1}} = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}$$

$$C_{n-1} = \frac{-1}{b_{n-1}} \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_{n-1} & b_{n-3} \end{vmatrix}$$

- Thee rouths Criteria States that the no of roots of $q(s)$ with (+ve) real part is equal to the no of change in sign of the first column.

So, the system is stable if there is no change in Sign of the first Column.

ex

2	1	3
4	5	6
7	8	4
1	2	1
2	2	2

Stable — there is no change in sign

2	2	3
2	1	2
5	-1	6
-6	7	2
-1	5	6

Unstable — there is change in sign

2	21
1	11
-1	21
2	2
1	

Unstable — there is change in sign

-1	5	3
-2	1	2
-1	1	1
-4		

Stable — because there is no change in sign

CASE 1 No element in the 1^{st} Column is Zero

Ex Check the stability of the 2^{nd} order system represented by $q(s) = as^2 + bs + c$

S^2	a	c
S	b	0
1	$\frac{--bc}{b} = \boxed{c}$	

The system is stable when :-

① $a>0 \quad b>0 \quad c>0$

the system is stable for all values of a, b, c is over than zero (all Positive)

or

② the system is stable for all values of a, b, c is less than zero (all negative)

$a<0 \quad b<0 \quad c<0$

Ex $q(s) = a_3 S^3 + a_2 S^2 + a_1 S + a_0$

S^3	a_3	a_1
S^2	a_2	a_0
S^1	$\frac{a_2 a_1 - a_3 a_0}{a_2}$	0
1	a_0	

If ~~Stable~~ ① all +ve

$$\frac{a_2 a_1 - a_3 a_0}{a_2} > 0$$

$$\boxed{a_2 a_1 > a_3 a_0}$$

① Stable if
$a_3 > 0$
$a_2 > 0$
$a_0 > 0$
$a_2 a_1 > a_3 a_0$

② if all −ve

Stable if

$a_3 < 0$

$a_2 < 0$

$a_0 < 0$

$a_2 a_1 > a_3 a_0$

$$\frac{a_2 a_1 - a_3 a_0}{a_2} < 0$$

when multiply $a_2 * 0$

$a_2 < 0$

so the sign $<$ become $>$

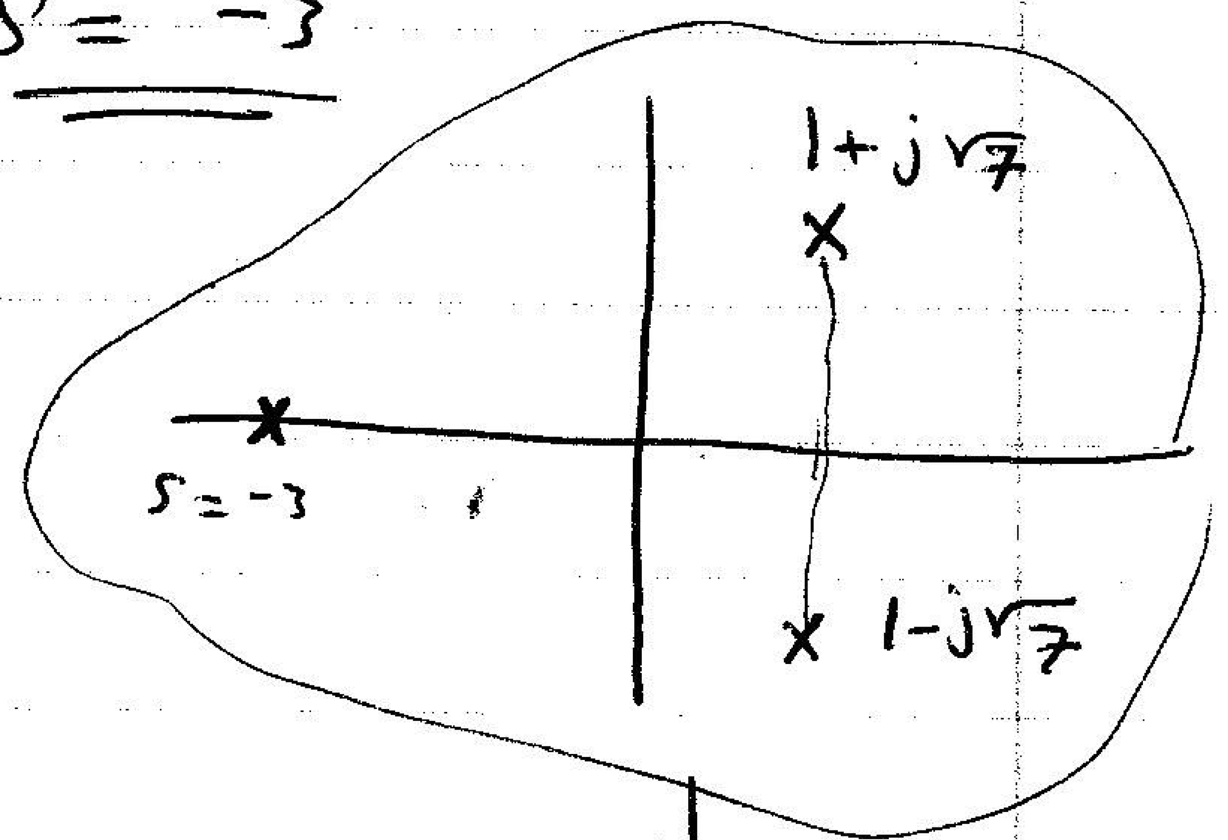
$\therefore a_2 a_1 > a_3 a_0$

Ex $s^3 + s^2 + 2s + 24$

Sol $s^3 + s^2 + 2s + 24 = 0$

$(s - 1 + j\sqrt{7})(s - 1 - j\sqrt{7})(s + 3) = 0$

$s = 1 - j\sqrt{7}$ $\quad s = 1 + j\sqrt{7}$ $\quad s = -3$



Unstable

Solve by rouths :-

s^3	1	2
s^2	1	24
s^1	−22	0
1	24	

root, root

∴ The system is Unstable!

CASE 2 : There is a zero in the 1^{st} Column, but Some other elements of the row Containing Zero are non-zero

⊛ If there is only one Zero after completing the array it might be replaced by [ε]

ε : is a small positive number approaches to Zero (0^+)

1	2	3
0	2	4
5	6	3

(ε : Epsilon)

EX Consider the following characteristic polynomial :-

$$q(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$$

s^5	1	2	11
s^4	2	4	10
s^3	$\varepsilon \to 0^+$	6	0
s^2	$\frac{-12}{\varepsilon}$	10	0
s	6	0	
1	10		

Cell 4X1

$$\frac{4\varepsilon - 12}{\varepsilon} = \frac{-12}{\varepsilon}$$

Cell 4X2

$$\frac{10\varepsilon - 0}{\varepsilon} = 10$$

Cell 5X1

$$\frac{\frac{-12}{\varepsilon} * 6 - 10\varepsilon}{\frac{-12}{\varepsilon}} * \frac{\varepsilon}{\varepsilon}$$

$$= \frac{-72 - 10\varepsilon}{-12}$$

$$= \frac{-72 - 0}{-12} = 6$$

Cell 5X2

$$\frac{\frac{-12}{\varepsilon} * 0 - 0 \times \varepsilon}{\frac{-12}{\varepsilon}} = 0$$

∴ The system is Unstable !

2 +ve roots

2 change in sign

ex $q(s) = s^4 + s^3 + s^2 + s + k$

s^4	1	1	k
s^3	1	1	0
s^2	ϵ	k	0
s^1	$\frac{\epsilon - k}{\epsilon}$	0	0
1	K		

⊛ To be stable :-

$\frac{\epsilon - k}{\epsilon} > 0 \Rightarrow \frac{-k}{\epsilon} > 0 \Rightarrow -k > 0 \Rightarrow \boxed{k < 0}$

$\boxed{K > 0}$

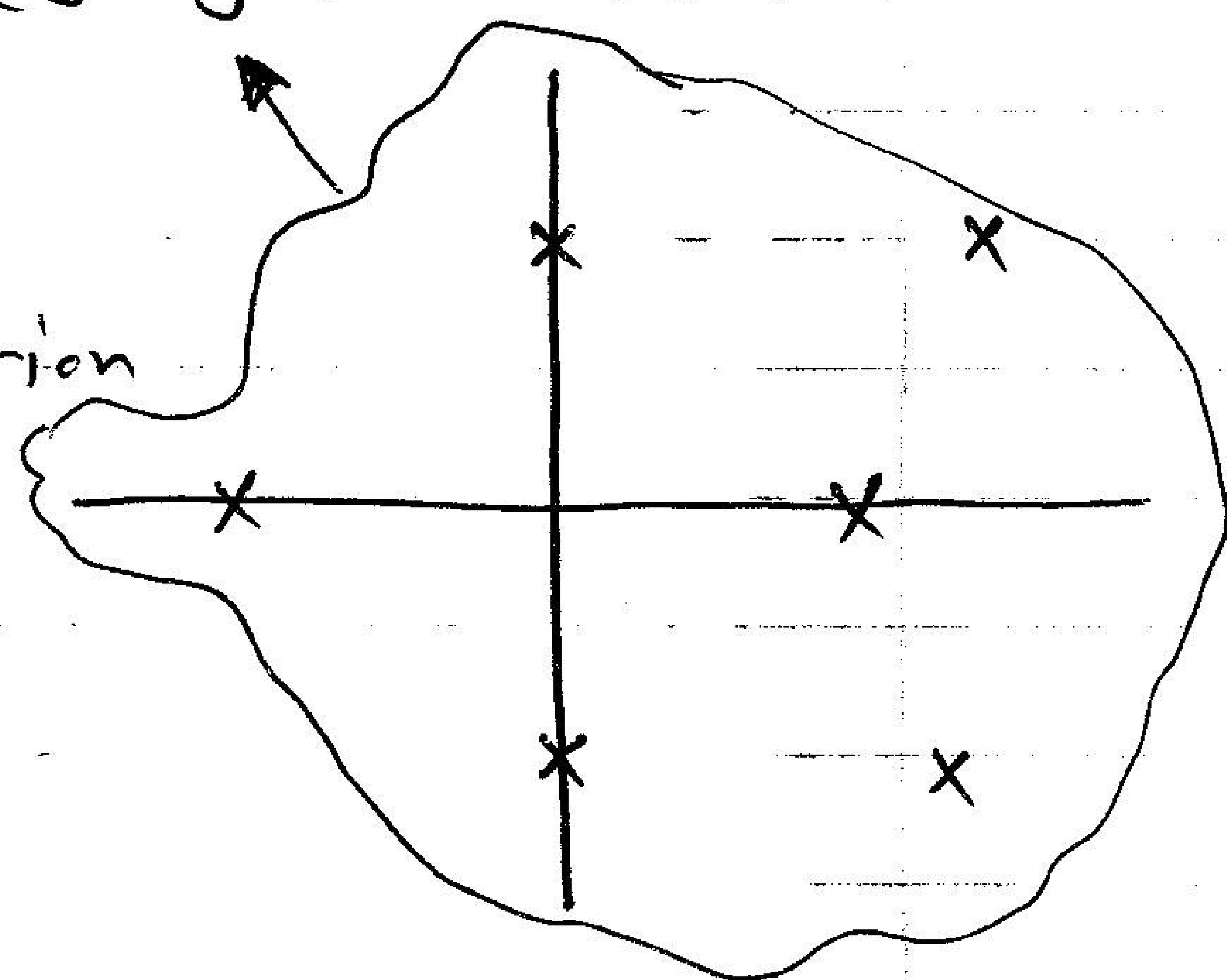
∴ The system is Unstable for any Value of k except for $k = 0$, the system then is said to be marginally stable.

CASE 3 : There is a zero in the 1st Column, and other elements of the row Containing zero are zeros

* This condition occurs when polynomial contains singularities that are symmetrical allocated about the origin.

There for, a factors such as :

$(S+\sigma)(S-\sigma)$ or $(S+jw)(S-jw)$ occur.



* by utilizing the auxiliary equation $U(S)$: which is immediately Precedes the zero entry in Routh array

---

Ex : Consider the following polynomial

$$S^3 + 2S^2 + 4S + K$$

S^3	1	4
S^2	2	K
S^1	$\frac{8-K}{2}$	
1	K	

To be stable :-

$K > 0$ $\frac{8-K}{2} > 0$

$\frac{8-K}{2} > 0 \Rightarrow 8-K > 0$

$\Rightarrow K < 8$

$\Rightarrow \therefore 0 < K < 8$

• but if $k = 8$:-

$$\frac{8-k}{2} \Rightarrow \frac{8-8}{2} = \frac{0}{2} = \boxed{0}$$

s^3	1	4
s^2	2	k
s^1	0	0
1	k	

The Auxiliary equation (The row above the zeros)

$$U(s) = 2s^2 + k = 0 \longrightarrow \; s^2 \mid 2 \quad k$$

$$\Rightarrow 2s^2 + 8 = 0$$

$$s^2 + 4 = 0 \Rightarrow \boxed{s = \pm 2j}$$

Note :- The Auxiliary equation must be s^2, s^4, s^6 ---

The power of s must be <u>even</u>

الاحد
11/17

Ex :- Consider the characteristic polynomial & find the intersection point with Imaging axis :-

$$q(s) = s^5 + s^4 + 4s^3 + 24s^2 + 3s^1 + 63$$

s^5	1	4	3
s^4	1	24	63
s^3	-20	-60	0
s^2	+21	63	0
s^1	0	0	0
1			

$\frac{1}{-20}(-20 * 24 - -60)$
$= (-480 + 60)\frac{1}{-20}$
$= +420 * \frac{1}{+20} = \boxed{21}$

$\frac{-20 * 63 - 1 * 0}{-20}$
$= \frac{-20 * 63}{-20} = \boxed{63}$

$\frac{(21)(-60) - (-20)(63)}{21}$
$= \frac{-1260 - -1260}{21}$
$= \frac{-1260 + 1260}{21}$
$= \frac{0}{21} = \boxed{0}$

Auxiliary Equation :-

$$21s^2 + 63 = 0$$

$$21s^2 = -63 \Rightarrow s^2 = \frac{-63}{21} = -3$$

$$\boxed{\therefore s = \pm\sqrt{3}\, j}$$

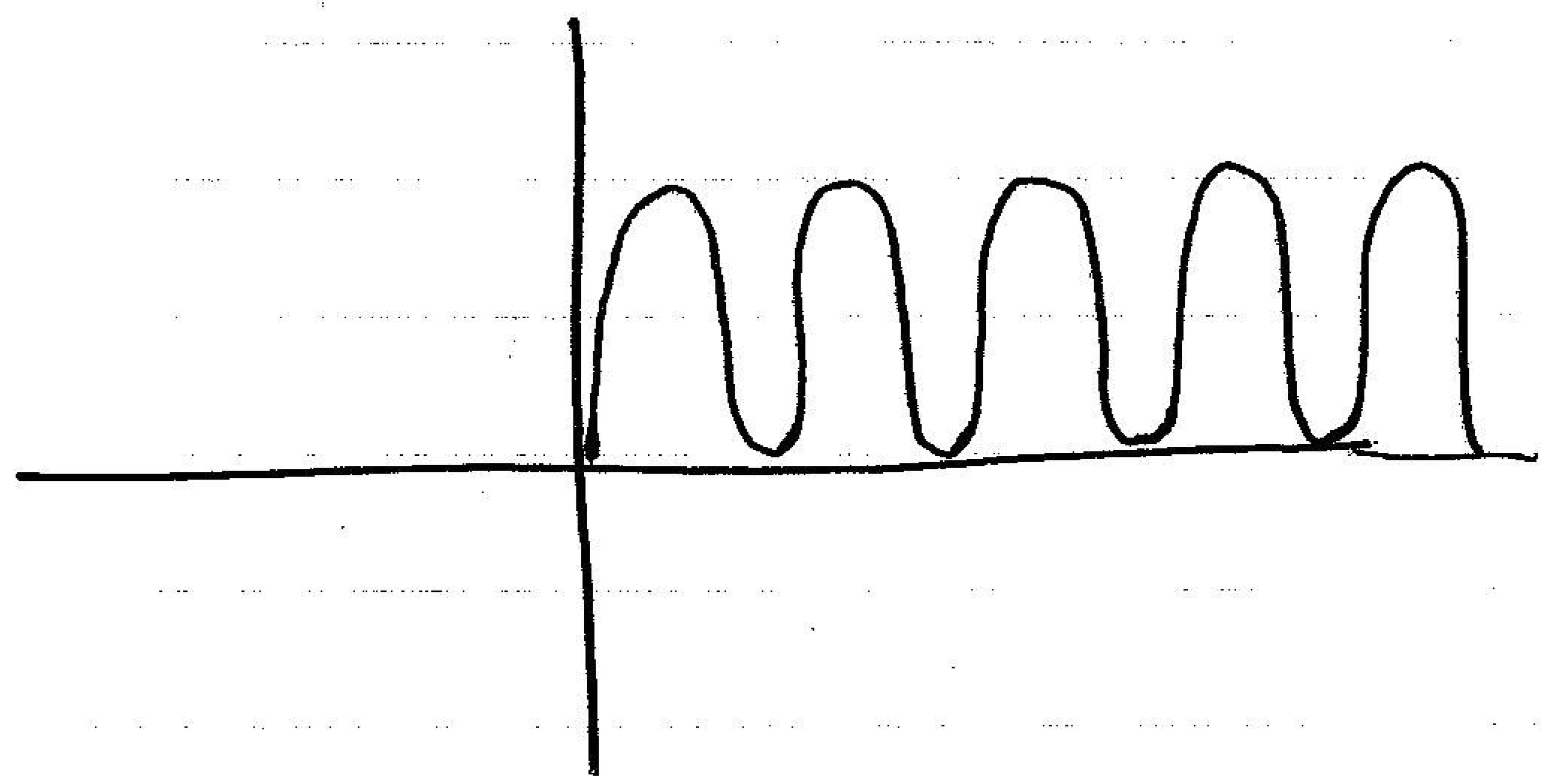
CASE 4 : Repeated Roots of the ch. equ. on the (jw) axis

* If the (jw) axis roots are simple, the system is neither stable Nor Unstable.

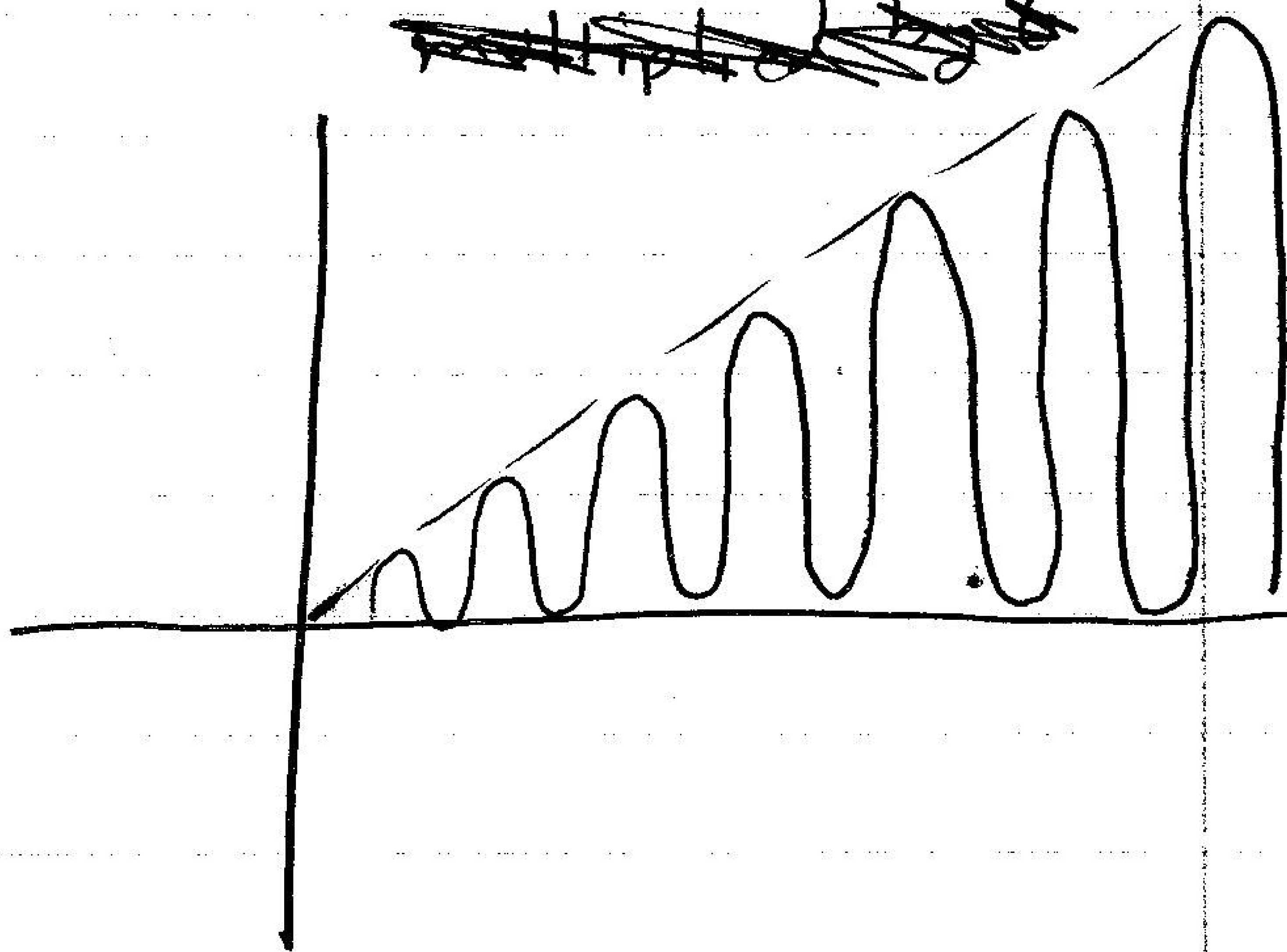
* Its instead Called marginally stable, since

it has an Undamped sinusoidal.

* If the (jw) axis roots are repeated, the system response will be __Unstable__, with a form of $\boxed{t\sin(\omega t+\theta)}$, thus Rouths table Cannot reveal this form of unstability.



Undamped Sinusoidal



when multiply the Undamped Sinusoidal by (t)

Ex $q(s) = S^5 + S^4 + 2S^3 + 2S^2 + S + 1$

S^5	1	2	1
S^4	1	2	1
S^3	ϵ	ϵ	0
S^2	1	1	0
S^1	ϵ	0	0
1	1		

$\frac{2\epsilon-\epsilon}{\epsilon} = \frac{\epsilon}{\epsilon} = \boxed{1}$

$\frac{\epsilon-0}{\epsilon} = \boxed{1}$

(:) In this case, the Routh table failed to reveal If the system is unstable or not

(*) So, we must take the Auxiliary equation :-

s^4	1	2	1

$$s^4 + 2s^2 + 1 = 0$$

$$(s^2+1)(s^2+1) = 0$$

$$\downarrow \qquad \qquad \searrow$$

$$\boxed{s = \pm j} \qquad \boxed{s = \pm j}$$

∴ The system is Unstable, due to ~~root duplicati~~

Imaginary root duplication

(*) imaginary part is repeated
Zero is repeated → the system is Unstable

Ex $q(s) = s^3 + s + 7$

s^3	1	1
s^2	ϵ	7
s^1	$\frac{-7}{\epsilon}$ → $-\infty$	0
1	7	

$$\frac{\epsilon - 7}{\epsilon} \Rightarrow \frac{-7}{\epsilon}$$

the system is unstable

Ex $q(s) = s^2 + s + 1$

s^2	1	1
s	1	0
1	1	

∴ the system is stable

Ex $q(s) = s^2 + 1$

s^2	1	1
s	ϵ	0
1	1	

$$\frac{(\epsilon)(1) - (1)(0)}{\epsilon} = \frac{\epsilon}{\epsilon} \neq 1$$

Stable

or marginally stable (أدقّ)

Ex $q(s) = s^2 - 1$

s^2	1	-1
s^1	ϵ	0
1	-1	

$$\frac{-\epsilon - 0}{\epsilon} = \frac{-\epsilon}{\epsilon} = \boxed{-1}$$

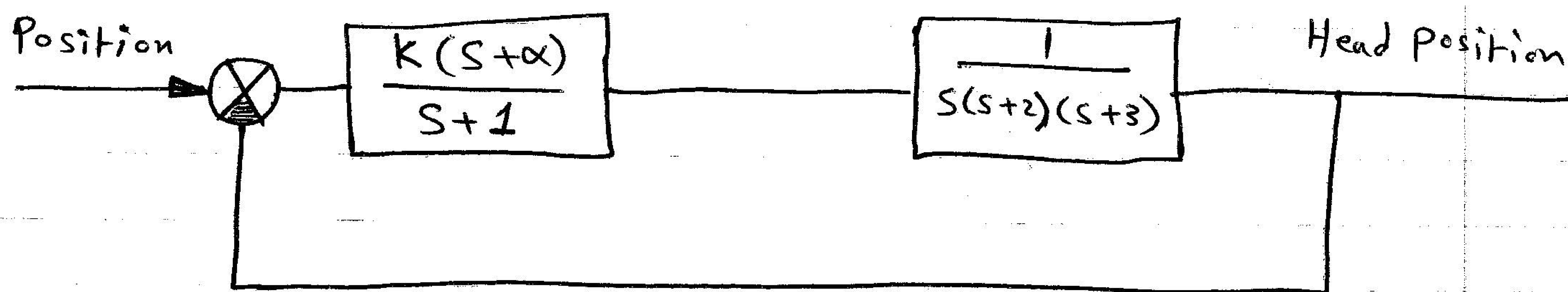
Unstable (there is a change of sign)

-

<u>Ex</u>: a large welding Robots are used in todays auto plants. the welding head is moved to different positions on the auto body, and a rapid accurate response is required.

* a block diagram of the welding head position is shown below.

* We must desire to determine the range of <u>K</u> and <u>α</u> for which the system is <u>stable</u>.



<u>Solution</u> :-

$$T.F. = \frac{\dfrac{K(S+\alpha)}{(S+1)(S)(S+2)(S+3)}}{1+\dfrac{K(S+\alpha)}{(S+1)(S)(S+2)(S+3)}}$$

$$= \frac{K(S+\alpha)}{(S+1)(S)(S+2)(S+3)+K(S+\alpha)}$$

$q(S)$ [المقام] $= S^4 + 6S^3 + 11S^2 + (K+6)S + K\alpha = 0$

ملاحظة هامة :- $q(s)$ هو مقام الـ T.F

s^4	1	11	$K\alpha$
s^3	6	$K+6$	0
s^2	$\frac{60-K}{6}$	$K\alpha$	0
s^1	B_3	0	0
1	$K\alpha$		

$$\frac{66-(K+6)}{6} = \frac{66-K-6}{6} = \boxed{\frac{60-K}{6}}$$

$$\frac{6K\alpha - 0}{6} = \boxed{K\alpha}$$

$$\frac{\frac{60-K}{6}(K+6) - 6K\alpha \cdot \frac{\times 6}{6}}{\frac{60-K}{6}} = \boxed{\frac{(60-K)(K+6) - 36K\alpha}{60-K}} = \boxed{B_3}$$

CASE 1 > 0

The system is stable when:-

$\boxed{\frac{60-K}{6} > 0}$

$\boxed{B_3 > 0}$

$\boxed{K\alpha > 0}$

① $\frac{60-K}{6} > 0$

$\Rightarrow 60 - K > 0$

$\Rightarrow 60 > K \Rightarrow \boxed{K < 60}$

② $K\alpha > 0$

$\therefore$ $K > 0$, $\alpha > 0$

1

③ $B_3 > 0$

$$\frac{(60-k)(k+6) - 36k\alpha}{60-k} > 0$$

$$(60-k)(k+6) - 36k\alpha > 0 \qquad \times(-)$$

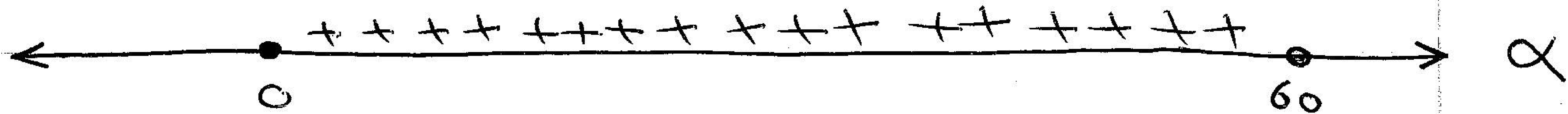
$$(k-60)(k+6) + 36k\alpha \leq 0$$

$$(k-60)(k+6) \leq -36k\alpha$$

$$\frac{(k-60)(k+6)}{-36k} \leq \frac{-36k\alpha}{-36k}$$

عند القسمة على سالب
تنعكس اشارة المتباينة

$$\Rightarrow \frac{(60-k)(k+6)}{36k} \geq \alpha$$



∴ The system is stable when :-

① $0 < k < 60$ [k is positive].

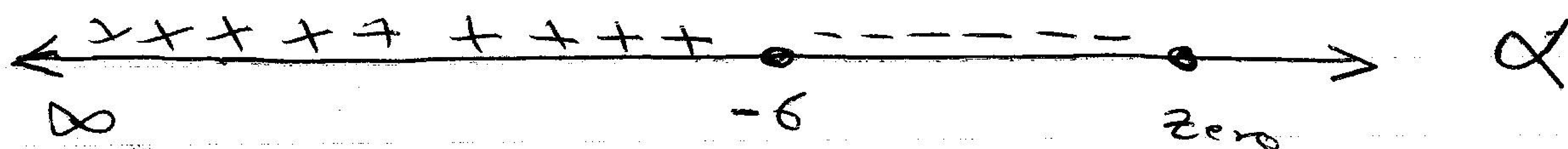
② $\alpha \leq \dfrac{(60-k)(k+6)}{36k}$ [α is positive].

CASE 2 <0

System is stable when

$K<0$ $\alpha<0$

$$\frac{(60-K)(K+6)}{36K} \geq \alpha$$



∴ The system is stable when :-

① $-6 < K < 60$

② $\alpha \leq \dfrac{(60-K)(K+6)}{36K}$

إلى هنا مادة الفيرست ...

Until Here ... The 1st Exam Material...

"بالتوفيق للجميع

إن شاء الله ☺ "